Proceedings Title BSSPP
Proceedings BSSPP Series No. xxx, 2006
A.C. Editor, B.D. Editor & C.E. Editor, eds.

DOI: 00.0000/X000000000000000X

Heating of solar and astrophysical plasmas by Burgulence and Alfvén waves

T. M. Mishonov[1], Y. G. Maneva[1], I. I. Roussev[2], and T. S. Hristov[3]

[1]Department of Theoretical Physics, University of Sofia, 5 J. Bourchier Blvd, BG-1164 Sofia, Bulgaria;
e-mails: mishonov@phys.uni-sofia.bg, yanamaneva@phys.uni-sofia.bg [2]Institute for Astronomy, 2680 Woodlawn Dr, Honolulu, HI 96822, USA;
e-mail: iroussev@ifa.hawaii.edu [3] Department of Earth and Planetary Sciences, Johns Hopkins University, 3400 N. Charles St, Baltimore, MD 21218, USA;
e-mail: tihomir.hristov@jhu.edu

Abstract. We present a new theoretical framework to explain the heating of solar and astrophysical plasmas, which invokes the Burgers approach to turbulence as applied to the Alfvén waves. In our approach, the Alfvén waves are considered as intermediary between the turbulent energy and the heat. We employ the Burgers turbulence to calculate the energy flux of Alfvén waves along the mean magnetic field in the plasma. The obtained results are relevant to the wave channel of heating of the solar corona. By incorporating the dissipation of plasma waves instabilities, the suggested heating model has the potential of describing not only the coronal heating, but also the abnormal viscosity in accretion discs as well as the anomalous properties of other astrophysical plasmas. We argue that the suggested Langevin–Burgers approach in describing turbulence can be useful in other physical circumstances in which there is a strong interaction between a stochastic system and waves. Furthermore, the proposed methodology can be applied to a vast number of multidisciplinary researches involving hydrodynamics and magnetohydrodynamics.

Keywords. MHD–magnetic fields–turbulence–waves–Sun: corona–accretion disks

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1. Introduction

For over 60 years now, we are facing the perplexing astrophysical problem of why the temperature of the solar corona is by two orders of magnitude greater than that of the photosphere (see Mandrini, Démoulin, & Klimchuk 2000 and the references therein). The purpose of the present work is to investigate the wave mechanism of heating of the solar corona, according to which energy is being transported outward by magnetohydrodynamical (MHD) waves generated in the stochastic convective zone. Our main idea is similar to that about the granular origin of the kink modes – carrying photospheric energy into the corona – suggested by recent observations Kukhianidze, Zaqarashvili, & Khutsishvili (2006). In the framework of this scenario, the well established correlation between the solar activity and the X-ray emission of the corona can be easily explained. The novelty in our model is that we adopt the Burgers approach to the turbulence Polyakov (1995) to calculate the spectral density of Alfvén waves. In the framework of this model, we derive explicit formulas for: (1) the spectral density of MHD waves; (2) the total energy density; and (3) the dissipated wave power per unit volume. In perspective, these derived quantities can be generalized for any realistic model of turbulence and distributions of mass density, temperature, and magnetic field in the plasma. In the current work, the model task of calculating the wave power emitted by a turbulent halfspace and transmitted in a non-moving magnetized plasma is considered as an illustrative example.

1.1. *Burgulence*

The Burgers approach to the turbulence Burgers (1948) provides an opportunity for approximate treatment of a multitude of problems; as a tutorial on "Burgulence" and a source of main references see Frish & Bec (2001). This approach has been introduced to the astrophysics community by Zeldovich (1970), and later on used by Arnold, Shandarin, & Zeldovich (1982). These works, however, were not fully appreciated until recently. In the last decade, the development of the quantum field theory has given incitement to the theory of turbulence as well. The Kolmogorov power laws Kolmogorov (1941) have been derived and gradually the Burgers approach converted from a sophisticated high-energy physics Polyakov (1995) to a standard tool for probing the nature of turbulence over different physical phenomena. The concept of turbulent convection in the photosphere as a random driver for coronal heating was explored by a number of researchers Abramenko, Pevtsov, & Romano (2006), Dahlburg, Klimchuk, & Antiochos (2005), Gudiksen & Nordlund (2002), Priest, Heyvaerts, & Title (2002). The turbulence induces random motions of foot-points of magnetic flux tubes. As a result, MHD waves are generated at the upper convection zone, which propagate upward in the corona along the magnetic field. The wave energy is ultimately deposited to the plasma as heat, which is believed to be sufficient to maintain the corona at the observed million-degree state. The concept of a random driver was also invoked to explain other solar phenomena, for example spicules Erdélyi & James (2004), Selwa, Skartlien, & Murawski (2004).

The stochastic behavior of convective cells in the solar convective zone resembles the chaotic nature of vortices in the turbulent hydrodynamics. Therefore, under our considerations, the influence of the stochastic velocity on the convective zone can be simulated by models, which can then be used to describe turbulence. Among the existing models, the Burgers approach is the simplest possible one that allows analytical treatment. An application of chaotic models to the solar-terrestrial environment was recently made by Chian et al. (2006). As mentioned in that study, the nonlinear spatial-temporal evolution of Alfvén waves can be modelled by the nonlinear Schrödinger differential equation, which in theoretical physics is also known as the time-dependent Ginzburg–Landau (TDGL) equation. This equation is also in the basis of superconductivity physics, and especially in the theory of fluctuational superconductivity. This similarity serves to identify common problems from very distant fields of physics, like astrophysics and condensed matter physics. This may result into the appearance of some interdisciplinary research work focused on exploring the influence of random drivers over deterministic wave systems.

In the next section, we will consider the application of the Burgers approach to the theory of generation of Alfvén waves from the turbulence and the stochastic granulation.

2. Wave Amplitudes and Spectral Densities

According to the pioneering work of Burgers (1948), the influence of turbulent vortices on the fluid motion can be modelled by introducing a random volume density of an external force, $\mathbf{F}(t, \mathbf{r})$, on the right-hand side of the equation of motion in magnetohydrodynamics (MHD):

$$\rho(\partial_t + \mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p + \eta \Delta \mathbf{V} + \mathbf{j} \times \mathbf{B} + \mathbf{F}. \tag{2.1}$$

All the notations used here have their usual meaning. By analyzing low-frequency and long-wavelength phenomena, one can approximate the random force, $\mathbf{F}$, to be a white noise with a δ-function correlator given by Burgers (1948), Frish & Bec (2001), Polyakov (1995), Zeldovich (1970), Arnold, Shandarin, & Zeldovich (1982):

$$\langle \mathbf{F}(t_1,\mathbf{r}_1)\mathbf{F}(t_2,\mathbf{r}_2)\rangle = \tilde{\Gamma}\rho^2\delta(t_1-t_2)\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbb{1}, \tag{2.2}$$

where $\tilde{\Gamma}$ is the Burgers parameter. In Eq. 2.1, the current density, $\mathbf{j}$, is given by Ohm's law

$$\mathbf{j} = \sigma\mathbf{E}', \quad \mathbf{E}' = \mathbf{E} + \mathbf{V}\times\mathbf{B}, \tag{2.3}$$

where σ is the electrical conductivity and $\mathbf{E}'$ is the effective electric field acting on the fluid. In the quasi-stationary approximation ($\partial_t\mathbf{E} \approx 0$), the evolution of the magnetic field is governed by

$$\partial_t\mathbf{B} = \mathrm{rot}\,(\mathbf{V}\times\mathbf{B}) - \nu_{\mathrm{m}}\mathrm{rot}\,\mathrm{rot}\,\mathbf{B}. \tag{2.4}$$

Here we consider an incompressible fluid ($\mathrm{div}\mathbf{V} = 0$), which is a good approximation for the solar convection zone and the photosphere. We also assume small wave amplitudes of the velocity

$$\mathbf{V} = (V_x, V_y, V_z), \quad V \ll V_{\mathrm{A}}, \tag{2.5}$$

and the magnetic field

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}', \quad \mathbf{B}'(t,\mathbf{r}) = (B'_x, B'_y, B'_z), \quad B' \ll B_0, \tag{2.6}$$

compared to the Alfvén speed, V_{A}, and the external magnetic field, B_0, respectively. The z-axis in our geometry is chosen in the direction of $\mathbf{B}_0$: $\mathbf{B}_0 = (0,0,B_0) = B_0\mathbf{e}_z$. Analogously, for the pressure we also suppose small deviations from a constant value p_0: $p = p_0 + p'$. The linearized system of MHD equations then reads

$$\begin{aligned}
&\partial_t\mathbf{V} = -\frac{\nabla p'}{\rho} + \frac{\mathbf{F}}{\rho} + \frac{B_0}{\mu_0\rho}\begin{pmatrix}\partial_z B'_x - \partial_x B'_z\\ \partial_z B'_y - \partial_y B'_z\\ 0\end{pmatrix} + \nu_{\mathrm{k}}\Delta\mathbf{V},\\
&\partial_t\mathbf{B}' = B_0\partial_z\mathbf{V} + \nu_{\mathrm{m}}\Delta\mathbf{B}',\\
&\mathrm{div}\mathbf{B}' = 0, \quad \mathrm{div}\mathbf{V} = 0,
\end{aligned} \tag{2.7}$$

where ν_m and ν_k are the magnetic and kinematic viscosities, respectively.

Here we would like to emphasize that the incompressibility condition is applicable to the solar convective zone, where the sound speed significantly exceeds the Alfvén speed ($c_{\mathrm{S}} \gg V_{\mathrm{A}}$). In this region, the convective circulation is treated as the influence of turbulence in the Burgers approach. The excited Alfvén waves then propagate along the magnetic field lines and reach the corona, where the Alfvén speed becomes greater than the sound speed.

Now let us consider the evolution of the amplitudes of standing plane waves given by

$$\mathbf{V}(t,\mathbf{r}) = V_A\vec{v}_k(\tau)\sin(\mathbf{k}\cdot\mathbf{r}), \tag{2.8}$$

$$\mathbf{B}'(t,\mathbf{r}) = B_0\mathbf{b}_{\mathbf{k}}(\tau)\cos(\mathbf{k}\cdot\mathbf{r}), \tag{2.9}$$

$$\begin{aligned}
\mathbf{F}(t,\mathbf{r}) &= \rho V_{\mathrm{A}}\mathbf{f}_k(t)\sin(\mathbf{k}\cdot\mathbf{r}),\\
p' &= \rho V_{\mathrm{A}} p_k\cos(\mathbf{k}\cdot\mathbf{r}),
\end{aligned} \tag{2.10}$$

$$\langle \mathbf{f}^*_{\mathbf{p}}(t_1)\mathbf{f}_{\mathbf{k}}(t_2)\rangle = \Gamma\delta(t_1-t_2)\delta_{\mathbf{p},\mathbf{k}}\mathbb{1}. \tag{2.11}$$

The substitution of these expressions into Eq. 2.7 yields a separation of the variables.

Then, for the wave amplitudes, we obtain a system of ODEs of the kind

$$\mathrm{d}_t\vec{v} = p\,\mathbf{k} + f - V_\mathrm{A}\begin{pmatrix} k_z b_x - k_x b_z \\ k_z b_y - k_y b_z \\ 0 \end{pmatrix} - \nu_\mathrm{k} k^2 \vec{v}, \tag{2.12}$$

$$\mathrm{d}_t\mathbf{b} = V_\mathrm{A} k_z \vec{v} - \nu_\mathrm{m} k^2 \mathbf{b}, \tag{2.13}$$

$$\mathbf{k}\cdot\mathbf{b} = 0, \tag{2.14}$$

$$\mathbf{k}\cdot\vec{v} = 0. \tag{2.15}$$

Here, for the sake of brevity, the wave-vector indices are omitted. After taking a time derivative of Eq. 2.15 and then substituting the result into Eq. 2.12, we obtain an explicit form of the pressure:

$$p = -\frac{\mathbf{k}\cdot\mathbf{f}}{k^2} - V_\mathrm{A} b_z. \tag{2.16}$$

Further, the substitution of the pressure into Eq. 2.12 yields

$$\begin{aligned}\dot{\vec{v}} &= -V_\mathrm{A} k_z \mathbf{b} - \nu_\mathrm{k} k^2 \vec{v} + \mathbf{f}_\perp,\\ \dot{\mathbf{b}} &= V_\mathrm{A} k_z \vec{v} - \nu_\mathrm{m} k^2 \mathbf{b}.\end{aligned} \tag{2.17}$$

Here, $\mathbf{f}_\perp = \hat{\Pi}_\perp \cdot \mathbf{f}$ is the transverse projection of the external force with respect to the wave-vector, $\mathbf{k}$, by the polarization operator $\hat{\Pi}_\perp = \mathbb{1} - \mathbf{k}\otimes\mathbf{k}/k^2$. Now let us investigate the eigen-modes ($\propto \exp(-i\omega t)$), while discarding for a moment the influence of the external force, $\mathbf{f}$. For the case of small dissipation, the secular equations for all the components are the same

$$\begin{vmatrix} -i\omega + \nu_\mathrm{k} k^2 & -V_\mathrm{A} k_z \\ V_\mathrm{A} k_z & -i\omega + \nu_\mathrm{m} k^2 \end{vmatrix} = 0, \tag{2.18}$$

with eigen-values $\omega \approx \omega_\mathrm{A} - i\gamma/2$ (where $\omega_\mathrm{A} = V_\mathrm{A} k_z$ is the frequency of the Alfvén waves), and

$$\gamma = \nu k^2, \quad \nu \equiv \nu_\mathrm{m} + \nu_\mathrm{k} \tag{2.19}$$

being the attenuation coefficient. After some algebra the MHD system (see Eq. 2.17) transforms into an effective oscillator equation with friction under an external force

$$\ddot{\mathbf{b}} = -\nu k^2 \dot{\mathbf{b}} - \left(\omega_\mathrm{A}^2 + \nu_\mathrm{k}\nu_\mathrm{m} k^4\right)\mathbf{b} + \omega_\mathrm{A}\mathbf{f}_\perp, \tag{2.20}$$

which in the case of negligible ν has an effective energy

$$\mathcal{E} = \frac{1}{2}\left(\dot{b}^2 + \omega_\mathrm{A}^2 b^2\right). \tag{2.21}$$

A harmonic oscillator under an external force is a well-known mechanics problem Landau & Lifshitz (1990). When the external force is a white noise, a simple averaging gives the energy of the oscillator being a linear function of time; in other words, the random noise provides constant power to the oscillator

$$\mathrm{d}_t\langle\mathcal{E}\rangle = \mathrm{d}_t\left\langle \frac{1}{2}\dot{b}^2 + \frac{1}{2}\omega_\mathrm{A}^2 b^2 \right\rangle = \omega_\mathrm{A}^2 \mathrm{d}_t\left\langle b^2\right\rangle = \frac{3}{2}\Gamma. \tag{2.22}$$

In this equation,

$$\Gamma \equiv \frac{\tilde{\Gamma}}{\mathcal{V} V_\mathrm{A}^2} \tag{2.23}$$

is the Burgers parameter in the correlator for the plane-wave amplitudes of the transverse projection of the random force in Eq. 2.11, namely

$$\langle \mathbf{f}^*_{\mathbf{p},\perp}(t_1)\mathbf{f}_{\mathbf{k},\perp}(t_2)\rangle = \Gamma\delta(t_1 - t_2)\delta_{\mathbf{p},\mathbf{k}}\mathbb{1}. \tag{2.24}$$

Here the technical details of the derivation are omitted. We suppose that the wave energy does not increase significantly over one wave period ($\Gamma \ll \omega_{\mathrm{A}}$), and the brackets stand for averaging over wave period and noise. In steady-state, the condition for dynamic equilibrium requires that the power received from the noise (see Eq. 2.22) equals the dissipated power

$$\mathrm{d}_t\langle\mathcal{E}\rangle = -\gamma\langle\mathcal{E}\rangle. \tag{2.25}$$

This gives the averaged static energy of the oscillator

$$\mathcal{E}_{\mathrm{st}} = \frac{3\Gamma}{2\gamma} = \langle b^2\rangle_{\mathrm{st}}. \tag{2.26}$$

Now let us apply this model example to derive the spectral density of Alfvén waves. For the volume density of the average wave energy we have

$$\langle E\rangle = \frac{\langle B'^2\rangle}{2\mu_0} + \frac{\rho\langle V^2\rangle}{2} = \frac{1}{2}\langle b^2\rangle p_{B}, \tag{2.27}$$

where we have performed a volume averaging ($\langle\sin^2(\mathbf{k}\cdot\mathbf{r})\rangle = 1/2$), and used the definitions for the magnetic pressure, $p_B \equiv B_0^2/(2\mu_0)$, and Alfvén speed, $V_{\mathrm{A}}^2 = B_0^2/(\mu_0\rho)$. In Eq. 2.27, we can substitute the static squared amplitude $\langle b^2\rangle_{\mathrm{st}}$ from Eq. 2.26 and the attenuation coefficient from Eq. 2.19. Thus, for the static value of the average spectral energy density of the waves we finally get

$$E_k \equiv \langle E\rangle_{\mathrm{st}} = \frac{3}{4}\frac{\Gamma p_B}{\nu k^2}. \tag{2.28}$$

This spectral density is the main detail for the statistical analysis made in the next section.

3. Energy Density and Flux, and Heating Rate

In order to calculate the total static energy density of the waves, we have to perform a summation of the spectral density (see Eq. 2.28) over all allowed wave vectors

$$E_{\mathrm{tot}} = \mathcal{V}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}E_k. \tag{3.1}$$

The cut-off of the wave vectors is determined by the minimal size of the solar granulation in the convective envelope, r_{gr}: $k_c = 1/r_{\mathrm{gr}}$. Taking into account that the Alfvén waves are spreading axially symmetric as they follow the magnetic field lines, we may rewrite the wave-vector as $k^2 = k_z^2 + k_\rho^2$. This fixes the maximal value of the plane components of the wave-vector, k_ρ, as a function of k_c: $k_\rho^{(\max)} = \sqrt{k_c^2 - k_z^2}$. These cut-offs will be used as boundary conditions in the calculations of the total wave energy—the one integrated over all possible wave-vectors in Eq. 3.1 and the corresponding wave power. By recalling the axial symmetry of the problem we obtain

$$E_{\mathrm{tot}} = \frac{3\mathcal{V}\Gamma p_B}{(2\pi)^3\nu_{\mathrm{ph}}}\int_{k_z=\frac{1}{R}}^{k_c}\int_{k_\rho=0}^{k_\rho^{(\max)}}\frac{1}{k_z^2+k_\rho^2}\mathrm{d}(\pi k_\rho^2)\mathrm{d}k_z. \tag{3.2}$$

Here, we have accounted for the fact that, due to the "infrared divergence", the vertical component of the wave-vector should also have a minimal value determined by the typical size, R, of the given magnetic field loop, and $\nu_{\rm ph}$ stands for the viscosity at the photosphere. Then, having in mind Eq. 2.23, the final expression for $E_{\rm tot}$ becomes

$$E_{\rm tot} = \frac{3\rho_{\rm ph}\tilde{\Gamma}}{2\pi^2\nu_{\rm ph}R}\left[\ln\left(\frac{r_{\rm gr}}{R}\right) + \frac{R}{r_{\rm gr}} - 1\right] \approx \frac{3\rho_{\rm ph}\tilde{\Gamma}}{2\pi^2\nu_{\rm ph}r_{\rm gr}}. \quad (3.3)$$

In this approximation, we have taken into account that the size of sunspots significantly exceeds the granulation size: $R \gg r_{\rm gr}$. Thus, for the Alfvén waves energy flux along the magnetic field we obtain

$$S = \frac{1}{2}E_{\rm tot}V_{\rm A} \approx \frac{3\rho_{\rm ph}\tilde{\Gamma}V_{\rm A,\,ph}}{4\pi^2\nu_{\rm ph}r_{\rm gr}}. \quad (3.4)$$

The multiplication of the spectral density with the decay rate, γ, gives the heating rate in the corona. The summation of the spectral density in the wave-vector space results in the dissipated power of the Alfvén waves per unit volume

$$Q = \mathcal{V}\int E_k\gamma_k\frac{{\rm d}^3\mathbf{k}}{(2\pi)^3} = \frac{\tilde{\Gamma}\rho_{\rm cor}\nu_{\rm cor}}{16\pi^2r_{\rm gr}^3\nu_{\rm ph}}\frac{V_{\rm A,\,ch}}{V_{\rm A,\,cor}}. \quad (3.5)$$

Here, we have taken into account the observed difference in the values of kinematic viscosity at the photosphere, $\nu_{\rm ph}$ (where Alfvén waves are generated), and in the corona, $\nu_{\rm cor}$: $\nu_{\rm cor} \gg \nu_{\rm ph}$. Notice that there is no explicit dependence on the magnetic field in Eq. 3.5. This expression demonstrates that the correlation between the magnetic flux and X-ray luminosity of the corona Fisher et al. (1998) can be determined simply by the area of the magnetized region of the Sun's visible surface.

4. Discussion on Coronal Temperature Jump

An important ingredient of every coronal heating model is how naturally it can explain the temperature jump from the Sun's photosphere to the corona. Here we solved an oversimplified problem, and one may ask whether this mathematical exercise can be physically sound as applied to the Sun. In our model, we argue that the viscosity of the solar corona ($\rho_{\rm cor}\nu_{\rm cor} \propto T_{\rm cor}^{5/2}$) is the main property that determines the final result given by Eq. 3.5. There is a constant energy flux of MHD waves propagating trough the plasma along the magnetic field lines. However, the energy absorption is strongly dependent on the temperature: $Q = \alpha\tilde{\Gamma}T_{\rm cor}^{5/2}/r_{\rm gr}^3$, where α denotes the remaining multipliers from Eq. 3.5; for illustration purposes, in this model calculation we neglect the temperature dependence of the Alfvén speed.

Imagine for a moment the Sun without a hot corona. The mass density of the solar plasma decreases with increasing height, which, in turn, leads to a decrease in the thermal conductivity – determined perhaps by convection. At some height above the turbulent photosphere, the heat stability is lost and we have a positive temperature back-connection; the thermal conductivity is already weak, and thus small temperature increase leads to enhanced energy absorption of MHD waves, further increasing the temperature. The plasma heating is inevitable, until a new physical process – radiation – becomes important. Note that the volume power of black-body radiation is negligible at low temperatures, but it has a strong temperature dependence: $Q_{\rm BB} = \beta T^4$, where the coefficient β is a well-known function of temperature and pressure. A thermal equilibrium is established when the Alfvén wave heating is balanced by cooling due to black-body radiation. This condition yields the equilibrium coronal temperature in our model: $T_{\rm cor} = \left[(\alpha\Gamma)/(\beta r_{\rm gr}^3)\right]^{2/3}$. This demonstrates how important is the minimum volume of the turbulent cell: $r_{\rm gr}^3$. In the present estimate, we took a wave-vector cutoff corresponding to the minimum size of granulation: $k_{\rm c} = 1/r_{\rm gr}$. In the case of turbulent cascade, however, we have to generalize Burgers model by taking into account the wave-vector dependence on the spectral density of the turbulent random driver, $\tilde{\Gamma}(k)$. If we neglect the heat conductivity, the temperature jump will be very sharp – as sharp as can be an interface in weakly turbulent plasma.

5. Conclusions

We presented a new theoretical framework that has the potential of explaining the turbulent heating of solar and astrophysical plasmas stemming from Langevin–Burgers approach to the turbulence as applied to the Alfvén waves. We argued that this approach in describing turbulence can be useful in other physical circumstances in which there is a strong interaction between a stochastic system and waves. Furthermore, the presented methodology can be applied to a vast number of multidisciplinary researches involving hydrodynamics and MHD, for example the initial period of creation of ocean waves by a turbulent wind; the final nonlinear picture is analyzed in great details in Hristov, Friehe, & Miller (1998), Hristov, Friehe, & Miller (2003).

To date, the coronal heating problem has not been resolved satisfactory not due to the lack of interest, but because of the difficulties concerning the simultaneous collection of reliable observational data for all processes running on the Sun's surface. For the sake of determining the parameters of each model, as a rule, we are required to use indirect means. The model that we put forward on the arena suffers the same shortfall. In order to verify the model, it is necessary to know the velocity-velocity correlator in the horizontal direction or, which is equivalent, the correlator of the small stochastic perpendicular components of the magnetic field, among other quantities. In brief, apart from having a realistic model of turbulence, we also need realistic models for the rest of the physics. This naturally appeals for a broad interdisciplinary collaboration involving the whole spectrum of solar physics – from the statistical MHD to the X-ray luminosity of the solar corona. The verification and validation of the suggested model require realistic Monte Carlo (MC) simulations. The analytical calculations performed in the current work are only test examples. They are, however, indispensable for a more realistic treatment of the problem via future related MC simulations. At this point, we propose an oversimplified, yet solvable model, whose realistic generalization may lead to an adequate theory for the heating of the solar corona.

A natural continuation of the current investigation is to take into account the influence of the shear flow over the magnetized turbulent plasma. Preliminary numerical calculations of the behavior of the MHD waves in shear flow show a significant amplification of the slow magneto-sonic mode. The waves energy amplification is actually a transformation of the shear flow energy into wave energy, which ultimately dissipates into heat. This dissipation results in an effective viscosity in the shear flow, and the current scenario for heating of the solar corona may also happen to be the theory for the missing viscosity in accretion disks. Understanding the origin of anomalous viscosity in a magnetized turbulent plasma is rather important for the proper explanation of the shining mechanism of quasars, as well as for the angular momentum transport during the formation of compact astrophysical objects.

Acknowledgements

The authors thank D. Damianov, R. Erdélyi, E. Marsch, A. Rogava, I. Sokolov, S. Solanki, T. Zaqarashvili, and I. Zhelyazkov for useful discussions and comments on the manuscript. The research work of I. R. has been supported by the following NSF grants: ATM-0454469 (SHINE) and ATM-0456971 (FDSS).

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