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Temperature of accretion disks in semidetached binary stars

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Abstract. To provide the correct description of the gas flow structure in binary stars the consistent solution of gasdynamic and radiation transfer equations is required. Unfortunately, present-day computing facilities are not sufficient to solve this problem correctly, so the gasdynamic description is carried out with various simplifications of energy losses. The simplified approach requires the temperature of the considered object to be predetermined. The analysis of heating and cooling processes in accretion disks shows that for realistic parameters ($\dot{M} \simeq 10^{-12} \div 10^{-7} M_{\odot}/year$ and $\alpha \simeq 10^{-1} \div 10^{-2}$), the corresponding gas temperature in the outer parts of the disk is from $\sim 10^4$ K to $\sim 5 \times 10^5$ K. The forms of governing equations suited both for cold and hot accretion discs are presented.

Keywords. close binaries, accretion disks, hydrodynamics

1. Introduction

Semidetached binaries belong to the class of interacting stars, where one of the components fills its critical surface, that results in mass transfer between the components of the system. In general, the shape of the critical surface is complex, and to describe mass transfer in the system, one should apply special mathematical models. However, observational data confirm that practically all binaries have circular orbits and their components rotate synchronously with the orbital motion. Therefore it is possible to simplify the problem and to determine the shape of a star by using the standard Roche model. In this case the critical surface coincides with an internal surface (the Roche surface) in a restricted problem of three bodies, and mass transfer between the components of the system occurs through the vicinity of the inner Lagrangian point L_1, where the pressure gradient is not counterbalanced by gravitation.

The gas dynamics of mass transfer through the inner Lagrangian point L_1 has been investigated by many authors. Prendergast (1960) and Gorbatsky (1964, 1977) first attempted to consider the gas flow structure in binaries. The application of numerical methods for studies of mass transfer in binaries was limited for a long time by insufficient computer power, and hence the two-dimensional models were widely used for the analysis of gas flow. In the last fifteen years gas dynamics of mass transfer was numerically studied with the help of more realistic 3D models (see, e.g. pioneering works by Nagasawa et al., 1991; Hirose et al., 1991; Molteni et al., 1991). The application of 3D numerical approaches to investigations of gas dynamics of mass transfer in close binaries has allowed us to obtain a body of new interesting results, that, to some extent, are changing standard point of view on flow patterns in semidetached binary systems (Bisikalo et al., 1998 - 2006; Molteni et al., 2001, Harmanec et al., 2002, Boyarchuk et al., 2002, Fridman et al., 2003, Kaigorodov et al., 2006).

To provide the correct description of the gas flow structure the consistent solution of gasdynamic and radiation transfer equations is required. Unfortunately, present-day computing facilities are not sufficient to solve this problem correctly, so the gasdynamic description is carried out with various simplifications of energy losses. Our models consider energy losses in the system by setting the adiabatic index $\gamma \sim 1$, i.e. we considered the near-isothermal case. The simplified approach requires the temperature of the considered object to be predetermined.

Let us consider a semidetached binary system with masses of accretor and mass-donating star M_1 and M_2 respectively, separation A, and angular velocity of orbital rotation Ω. Gaseous flows in the system can be described by Euler equations with addition of ideal gas equation of state. The shape of secondary is defined by Roche lobe geometry so there are only few dimensionless parameters determining the flow structure: mass ratio $q = M_2/M_1$, Lubow–Shu parameter $\epsilon = c(L_1)/A\Omega$ (Lubow & Shu, 1975) where $c(L_1)$ is sound speed in L_1, relative size of accretor R_1/A, and adiabatic index γ. We also should expand this list by adding the value of viscosity (expressed as dimensionless parameter α introduced by Shakura, 1972 and Shakura & Sunyaev, 1973) since in approximate solutions some numerical viscosity is present. The model implies that the magnetic and radiation fields do not influence the gas motion in the system. To solve the system of gasdynamic equations we use the Roe–Osher TVD scheme of a high approximation order with Einfeldt modification. This numerical method allows one to study flows with a significant density contrast, and, hence, to investigate the morphology of gaseous flows in binaries and consider the impact of the forming circumbinary envelope on the flow patterns.

The analysis of numerical results shows a sort of stability of the obtained solutions against perturbations of the input parameters when their values lie in intervals pertinent to cataclysmic variables (CVs) and low mass X-ray binaries (LMXB). The calculations prove the qualitative similarity in the structure of flow in the studied LMXB and CV systems. The gas dynamic structure of the flow is governed by the stream of matter from L_1, the accretion disk, the circumdisk halo, and the circumbinary envelope. The interaction between the stream and the disk is shockless, and the interaction of the matter of the circumdisk halo and circumbinary envelope with the stream leads to the formation of a shock with the form of a "hot line" along the edge of the stream. On the other hand, there are some differences in the solutions for disks with high and low temperatures. In particular, in solutions with hot and cold accretion disks there are spiral waves of different origins. It means, in turn, that the accretion disk temperature range in semidetached binaries is principal for numerical modeling. In this paper, we consider the temperature of an accretion disk for various accretion rates, i.e., the dependence $T(\dot{M})$ (Bisikalo et al., 2003), and briefly discuss the forms of the governing equations suited for the study of semidetached binaries.

2. Basic equations

The vertical structure of an accretion disk is determined by the balance between the vertical component of the gravitational force and the (vertical) pressure gradient, which, in turn, is specified by the balance between heating and cooling of the gas. The heating is associated with viscous dissipation of kinetic energy, and also with bulk radiative heating, which, in turn, is determined by the radiation of the central object. Cooling is brought about by several mechanisms: bulk radiative cooling, radiative heat conduction, and convection. Assuming that advective terms and terms associated with adiabatic heating or cooling are small, the steady-state energy equation

$$Q^+ - Q^- = 0$$

can be written as follows.

(1) For the optically thin case, when Q^+ is specified by bulk radiative heating and viscous heating and Q^- is determined by bulk radiative cooling,

$$Q^+_{visc}(\rho, T) + n^2 \cdot (\Gamma(T, T_{wd}) - \Lambda(T)) = 0\,. \tag{2.1a}$$

Here, $\Gamma(T, T_{wd})$ is the radiative-heating function, that depends on the gas temperature T and the temperature of the central object T_{wd}, $\Lambda(T)$ is the radiative-cooling function, and $Q^+_{visc}(\rho, T)$ is the viscous heating.

(2) For the optically thick case, Q^+ is specified by viscous heating, while Q^- is specified by radiative heat conduction (we neglect molecular heat conduction since it is very small compared with the radiative heat conduction) and convection in the vertical direction:

$$Q^+_{visc}(\rho, T) - \frac{\partial F_{rad}}{\partial z} - \frac{\partial F_{conv}}{\partial z} = 0\,. \tag{2.1b}$$

Here, F_{rad} and F_{conv} are the radiative and convective energy fluxes.

To determine the functions in (2.1a) and (2.1b), we will need

– the equation of continuity

$$-\dot{M} = 2\pi \int r \cdot \rho \cdot v_r \, \mathrm{d}z = \mathrm{const}\,, \tag{2.2}$$

– the equation of angular-momentum balance $\lambda \equiv r^2\Omega_K$ in the radial direction

$$\frac{\partial}{\partial r}\left(r \cdot \rho \cdot v_r \cdot \lambda\right) = \frac{\partial}{\partial r}\left(\nu \cdot \rho \cdot r^2 \cdot \frac{\partial \Omega_K}{\partial r}\right)\,, \tag{2.3}$$

from which it follows that

$$|v_r| = -\nu \cdot \Omega'_K \cdot \Omega_K^{-1} \cdot r^{-1} \simeq \nu \cdot r^{-1}\,, \tag{2.4}$$

– and the viscous heating

$$Q^+_{visc} = \rho \cdot \nu \cdot \left(r \cdot \frac{\partial \Omega_K}{\partial r}\right)^2\,. \tag{2.5}$$

Here, $\dot{M}$ is the accretion rate, $\Omega_K = \sqrt{GM/r^3}$ is the angular velocity of the Keplerian rotation of the disk, M is the mass of the central object, G is the gravitational constant, ρ is the density, v_r is the radial velocity, and ν is the coefficient of kinematic viscosity. Note that the molecular viscosity cannot provide the necessary dissipation, and dissipation processes are usually considered to be associated with turbulent or magnetic viscosity.

To determine the vertical pressure gradient, we will use the equation of hydrostatic balance in the vertical direction

$$\frac{1}{\rho} \cdot \frac{\partial P}{\partial z} = \frac{\partial}{\partial z}\left(\frac{GM}{\sqrt{r^2 + z^2}}\right) \simeq -\Omega_K^2 z\,, \tag{2.6}$$

as well as the equation of state of an ideal gas with radiation

$$P = \rho \mathcal{R} T + {}^1\!/_3 a T^4\,.$$

Here, P is the pressure, T is the temperature, $\mathcal{R}$R is the gas constant, and a is the radiation constant. All equations are given in cylindrical coordinates, (r, z).

3. The solution method

To determine the dependence $T(\overset{\bullet}{M})$, we will use (2.2) and (2.4) together with the expression for the viscosity coefficient ν. We will use the formula for ν suggested by Shakura (1972), $\nu = \alpha c_s H$, where H is the height of the disk and $c_s \simeq \sqrt{\mathcal{R}T + {}^1/_3 aT^4/\rho}$ is the sound speed. If we neglect the z dependence of the density and use $\bar{\rho}$ averaged over the height (further, we will denote this quantity simply as ρ), the integration of (2.6) yields the height of the disk H:

$$H = c_s \cdot \Omega_K^{-1} \,.$$

We will determine c_s from the temperature in the equatorial plane of the disk, $z = 0$. This approach is sufficiently correct for our purposes due to the uncertainty in the parameter α. As a result, we obtain an equation relating $\overset{\bullet}{M}$, $T|_{z=0}$ and ρ for the specified r and α,

$$\overset{\bullet}{M} = 2\pi \cdot \alpha \cdot \Omega_K^{-2} \cdot \rho \cdot c_s^3 = 2\pi \cdot \alpha \cdot \Omega_K^{-2} \cdot \left(\mathcal{R}T\rho^{2/3} + {}^1/_3 aT^4\rho^{-1/3}\right)^{3/2} \,. \qquad (3.1)$$

This equation reduces to a cubic equation in the variable $\rho^{1/3}$, and its solution has two branches: one with a negative real root and two complex ones, and one with three real roots, one of which is negative. Only positive real roots for the density are physically meaningful. For such roots to exist, the following condition must be satisfied:

$$\overset{\bullet}{M} > \frac{\sqrt{3}\pi \cdot \sqrt{\mathcal{R}} \cdot a \cdot \alpha \cdot T^{9/2}}{\Omega_K^2} \,, \qquad (3.2)$$

which yields the minimum accretion rate for the given T, r, and α. When deriving this condition, we used the equation of state taking into account the radiation pressure.

This estimate can also be written in the form

$$T < 7 \cdot 10^5 \left(\frac{r}{R_{wd}}\right)^{-2/3} \left(\frac{\overset{\bullet}{M}}{10^{-9} M_\odot/year}\right)^{2/9} \left(\frac{\alpha}{0.1}\right)^{-2/9} \quad \mathrm{K} \,,$$

where $R_{wd} = 10^9$ cm is the radius of the accretor (white dwarf).

Let us consider the condition (3.2) for the outer parts of the accretion disk. Let us take $r = A/5$, where A is the distance between the components of the binary ($A = 1.42R_\odot$), which corresponds to the situation for the dwarf nova IP Peg; as a result, we obtain

$$\overset{\bullet}{M} > 10^{-9} \left(\frac{T}{10^5\ \mathrm{K}}\right)^{9/2} \left(\frac{\alpha}{0.1}\right) \quad M_\odot/year \,. \qquad (3.3)$$

If (3.2) is satisfied, the roots of Eq. (3.1) relating $\overset{\bullet}{M}$, $T|_{z=0}$ and ρ for a given r and α can be written

$$\rho = (\mathcal{R}T)^{-3/4} \left(\frac{\overset{\bullet}{M}\Omega_K^2}{2\pi\alpha}\right) \sin^3 \left({}^1/_3 \arcsin \left(\sqrt{\mathcal{R}} a T^{9/2} \frac{2\pi\alpha}{\overset{\bullet}{M}\Omega_K^2} \right)\right) \,,$$

$$\rho = (\mathcal{R}T)^{-3/4}\left(\frac{\overset{\bullet}{M}\Omega_K^2}{2\pi\alpha}\right)\cos^3\left({}^1\!/_3 \arcsin\left(\sqrt{\mathcal{R}}aT^{9/2}\frac{2\pi\alpha}{\overset{\bullet}{M}\Omega_K^2}\right)+\frac{\pi}{6}\right)$$

(for simplicity, we have omitted numerical factors $\sqrt{3}/2 \simeq 1$). The first of these corresponds to disks with dominant radiation pressure ($\beta = {}^1\!/_3 aT^4/\rho\mathcal{R}T > 1$) and the second to disks with dominant gas pressure ($\beta < 1$).

These formulas describe the two branches of the two-parameter dependence $\rho(\overset{\bullet}{M}, T)$. To calculate the dependence $T(\overset{\bullet}{M})$, we must use the additional heat balance equation (1). As follows from Section 2, the form of (1) depends on the optical depth of the disk, which, accordingly, must be calculated.

4. Optical depth

The optical depth τ is specified by the product of the absorption coefficient κ, the density, and the geometrical depth of the layer, $\tau = \kappa \cdot \rho \cdot l$. For disk accretion, the basic parameter is the ratio of the geometrical depth of the layer where $\tau = 1$ and the height of the disk: $l^{\tau=1}/H$. After simple manipulation, we obtain

$$\frac{l^{\tau=1}}{H} = \frac{2\pi\cdot\alpha}{\kappa\cdot\overset{\bullet}{M}}\cdot c_s^2\cdot\Omega_K^{-1}\,. \tag{4.1}$$

The absorption coefficient κ displays a complicated dependence on T and ρ (and also on the degree of ionization, chemical composition, etc.). Here, we adopted the simple approximation for $\kappa(T,\rho)$ (Lin & Papaloizou, 1980; Alexander, Augason & Johnson, 1989; Bell & Lin, 1994)

$$\kappa(T,\rho) = \begin{cases} \kappa_1\cdot\rho^{2/3}\cdot T^3\,, & \kappa_1 = 10^{-8}\,, \quad (\kappa1)\\ \kappa_2\cdot\rho^{1/3}\cdot T^{10}\,, & \kappa_2 = 10^{-36}\,, \quad (\kappa2)\\ \kappa_3\cdot\rho\cdot T^{-5/2}\,, & \kappa_3 = 1.5\cdot 10^{20}\,, \quad (\kappa3)\\ \kappa_4\,, & \kappa_4 = 0.348\,. \quad (\kappa4)\end{cases}$$

According to Bell & Lin (1994), these four subregions correspond to scattering on molecular hydrogen, scattering on atomic hydrogen, freefree and freebound transitions, and Thompson scattering. The boundaries of the sub-regions, i.e., the transitions from one expression to another, are specified by the equality of the κ values calculated from these expressions. Figure 1 presents the dependences of κ on T and ρ. We can see regions with $d\kappa/dT > 0$, where thermal instability can develop when the dependence between the surface density and the disk temperature forms an S curve in the (Σ, T_{eff}) plane. Thermal instability is often invoked to explain the phenomenon of dwarf novae (see, for example, Cannizzo & Kenyon 1986; Meyer-Hofmeister & Ritter 1993); however, it is clear that this can occur only for sufficiently cool disks.

Let us return to (3.1), taking $\alpha = 0.1$ and $r = A/5$ and considering disks with dominant gas pressure, for which $\beta = {}^1\!/_3 aT^4/\rho\mathcal{R}T < 1$. The shaded region in the $(T, \overset{\bullet}{M})$ plane in Fig. 2a corresponds to all possible solutions for these disks. The dashed line corresponds to condition (3.3) for the existence of a solution for (3.1) there is no solution below

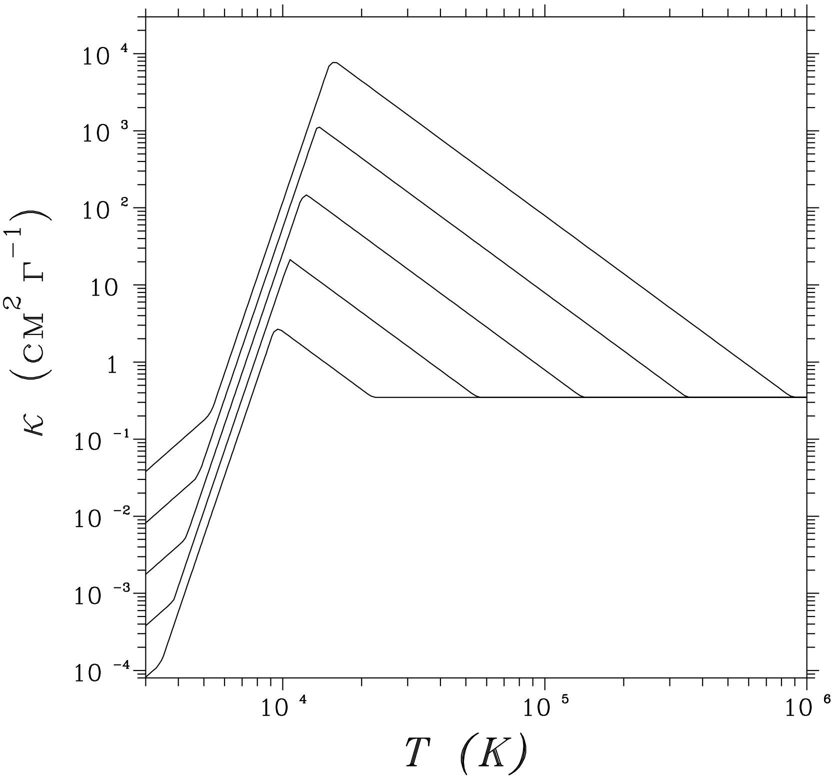

Figure 1. The $\kappa(T)$ dependence for $n = 10^{18}$ cm^{-3}, $n = 10^{17}$ cm^{-3}, $n = 10^{16}$ cm^{-3}, $n = 10^{15}$ cm^{-3}, $n = 10^{14}$ cm^{-3} (top to bottom, Bell & Lin 1994).

this line. The solid line indicates the boundary between the optically thick and optically thin solutions: the horizontal shading marks the region of optically thick disks, while the vertical shading marks optically thin disks. Figure 2b presents a similar pattern for disks with dominant radiation pressure ($\beta > 1$).

We can see from Fig. 2a that, for realistic values $\dot{M} \in [10^{-12}, 10^{-7}]M_\odot/yr$, disks with dominant gas pressure are mainly optically thick, though solutions corresponding to optically thin cool disks are possible for small $\dot{M}$. It follows from Fig. 2b that disks with dominant radiation pressure are mainly optically thin; optically thick hot disks can exist only for high $\dot{M}$.

5. Optically thick disks

In Section 3, we derived Eq. (3.1), which relates $\dot{M}$, T, and ρ for given r and α. Using the supplementary heat-balance equation (1), we can reduce the number of unknowns and obtain the desired relation between $\dot{M}$ and T.

The vertical temperature distributions in optically thick disks are described by the equation of radiative heat conduction with a source due to viscous heating (2.1b), which can be written in the form

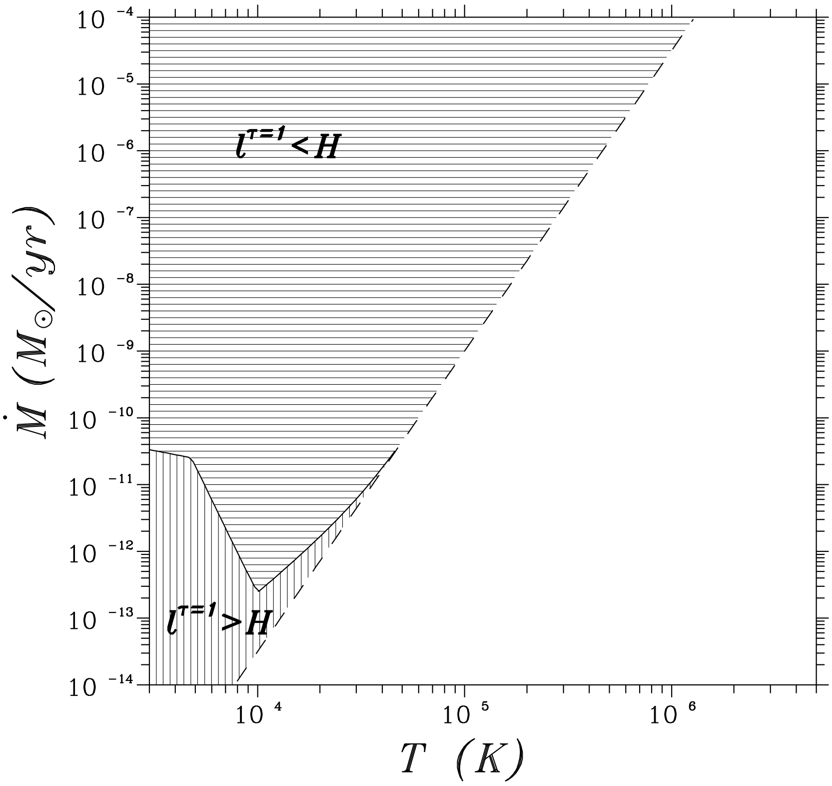

Figure 2a. Solutions of equation (3.1) for the gas dominated accretion disks on the $(T, \dot{M})$ plane for $\alpha = 0.1$ and $r = A/5$. Horizontal hatching shows optically thick disks and vertical hatching – optically thin disks. A solid line divides these two regions. A dashed line mirrors the condition (3.3) for the existence of the solution of equation (3.1) – there are no solutions below the line.

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z}\left(\frac{1}{\kappa\rho}\frac{\partial}{\partial z}\left({}^1\!/_3 acT^4\right)\right) + \rho\alpha c_s^2\Omega_K\,, \tag{5.1}$$

where e is the specific internal energy and c is the velocity of light. To solve (5.1), we must specify boundary conditions. Due to the symmetry of the problem, the temperature derivative in the equatorial plane must be zero; i.e., $T'|_{z=0} = 0$. The temperature at the upper boundary of the disk is specified by the condition $\Gamma(T_*, T_{wd}) = \Lambda(T_*)$. Though the functions $\Gamma(T, T_{wd})$ and $\Lambda(T)$ are complex, they are known and can be found in the literature (see, for example, Cox & Daltabuit 1971; Raymond, Cox & Smith 1976; Spitzer, 1978). The temperature derived by equating these functions (for a temperature of the central object (white dwarf) of $T_{wd} = 70000°$ K) is $T(H) = T_* = 13600°$ K.

The solution of (5.1) enters a stationary regime when the characteristic heat-conduction time

$$t_{diff} \simeq \frac{\mathcal{R}\kappa\rho^2 H^2}{acT^3}$$

is comparable to the time for viscous heating

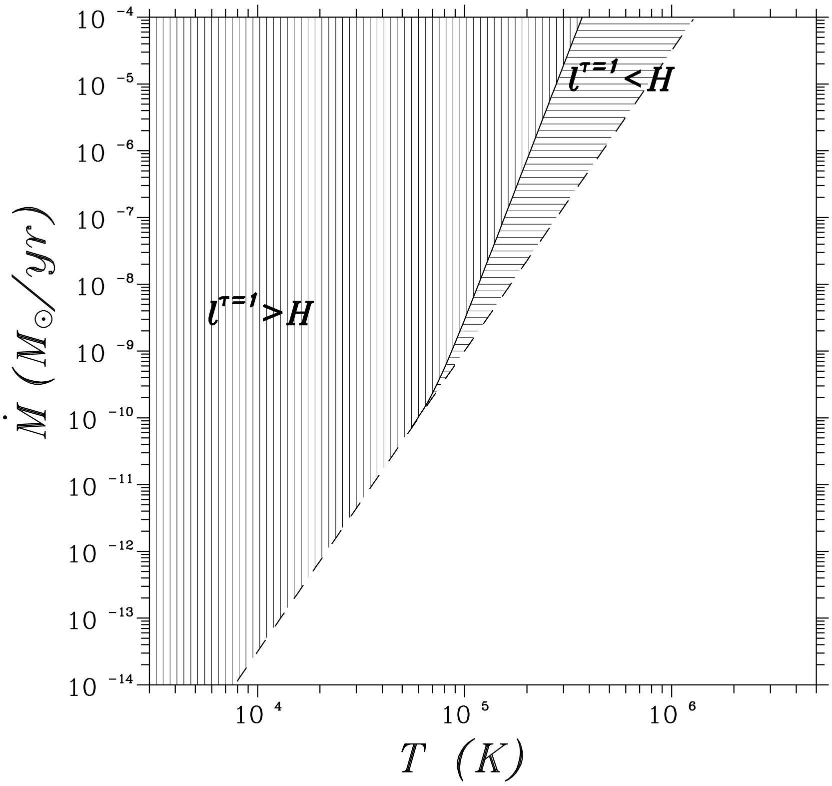

Figure 2b. The same as Fig. 2a but for radiation dominated disks.

$$t_{heat} \simeq \frac{\mathcal{R}T}{\alpha c_s^2 \Omega_K} \simeq \alpha^{-1}\Omega_K^{-1}\,.$$

Note that (5.1) can be integrated analytically in the steady-state case. Let us denote $U = T^4$, $U_* = T_*^4$, $U_0 = U|_{z=0}0$ and again assume that ρ does not depend on z. Then,

$$\frac{d}{dz}\left(\frac{1}{\kappa\rho}\frac{d}{dz}\left(\frac{ac}{3}U\right)\right) = -\rho\alpha c_s^2\Omega_K\,.$$

After integrating over z, we obtain

$$\frac{1}{\kappa\rho}\frac{d}{dz}\left(\frac{ac}{3}U\right) = -\rho\alpha c_s^2\Omega_K z\,.$$

The integration constant is equal to zero, since $U'|_{z=0} = 0$. For convenience, we will transform this last equation to the form

$$\frac{1}{\kappa}\frac{dU}{dz} \equiv \frac{dB}{dz} = -\frac{3}{ac}\rho^2\alpha c_s^2\Omega_K z\,,$$

where the function $B(U)$ is determined from the differential equation $\dfrac{dB}{dU} = \dfrac{1}{\kappa(U,\rho)}$ and can be written in an analytical form if ρ is fixed. Integrating this last equation over z, we obtain

$$B(U) = B(U_*) + \frac{3}{2ac}\rho^2\alpha c_s^2\Omega_K(H^2 - z^2)\,,$$

or, for $z = 0$,

$$B(U_0) = B(U_*) + \frac{3}{2ac}\rho^2\alpha c_s^2\Omega_K H^2\,.$$

Using the expressions

$$c_s^2 = \left(\mathcal{R}U_0^{1/4} + {}^1\!/_3\frac{aU_0}{\rho}\right)\,,$$

$$H^2 = \left(\mathcal{R}U_0^{1/4} + {}^1\!/_3\frac{aU_0}{\rho}\right)\Omega_K^{-2}$$

we obtain the algebraic equation for U_0

$$B(U_0) = B(U_*) + \frac{3}{2ac}\rho^2\alpha\Omega_K^{-1}\left(\mathcal{R}U_0^{1/4} + {}^1\!/_3\frac{aU_0}{\rho}\right)^2\,.$$

This equation implicitly specifies the dependence $U_0(\rho)$, i.e., $T(\rho)$. Expressing $\dot{M}$ in terms of ρ and T, we can derive the dependence $\dot{M}(\rho) = \dot{M}(T(\rho), \rho)$, which yields the dependence $T(\dot{M})$ in parametric form. Formally, the resulting solution can also exist in optically thin regions; however, given the adopted assumptions, these points can be rejected

Let us consider a graphical representation of the solution derived. Figure 3a presents the dependence $T(\dot{M})$ for $\alpha = 1$ and $r = A/5$, marked by asterisks. As in Fig. 2, the dashed lines bound from below the domain where solutions of (3.1) exist, while the solid line separates the domains of optically thin and optically thick disks. Figures 3b, 3c display the solutions for $\alpha = 0.1$ and $\alpha = 0.01$, respectively. Figure 3d presents the accretion rate as a function of the disk thickness. We can see that all the obtained disks are geometrically thin; i.e., $H \ll r$.

Radiative heat conduction is not the only mechanism for heat transfer into optically thin regions. Under certain conditions, convection can also play a substantial role. Neglecting the radiation pressure, the convective flux can be written in the form (Schwarzschild 1958; Bisnovatyi-Kogan 2001-2002)

$$F_{conv} = c_P \cdot \rho \cdot \left(\frac{|g|}{T}\right)^{1/2} \cdot \frac{l^2}{4} \cdot (\Delta\nabla T)^{3/2}\,,$$

$$\Delta\nabla T = -\frac{T}{c_P} \cdot \frac{\partial S}{\partial z}\,.$$

Here, c_P is the heat capacity at constant pressure, $S = \mathcal{R} \cdot \ln(T^{3/2}/\rho)$ is the specific entropy, $g = -\Omega_K^2 z$ is the gravitational acceleration, and l is the mixing length, taken to be $l = \alpha H$. To determine the vertical temperature distribution taking convection into account, we must solve the equation

$$\frac{\partial e}{\partial t} = \frac{\partial}{\partial z}\left(\frac{1}{\kappa\rho}\frac{\partial}{\partial z}\left({}^1\!/_3 acT^4\right)\right) - \frac{\partial F_{conv}}{\partial z} + \rho\alpha c_s^2\Omega_K \tag{5.2}$$

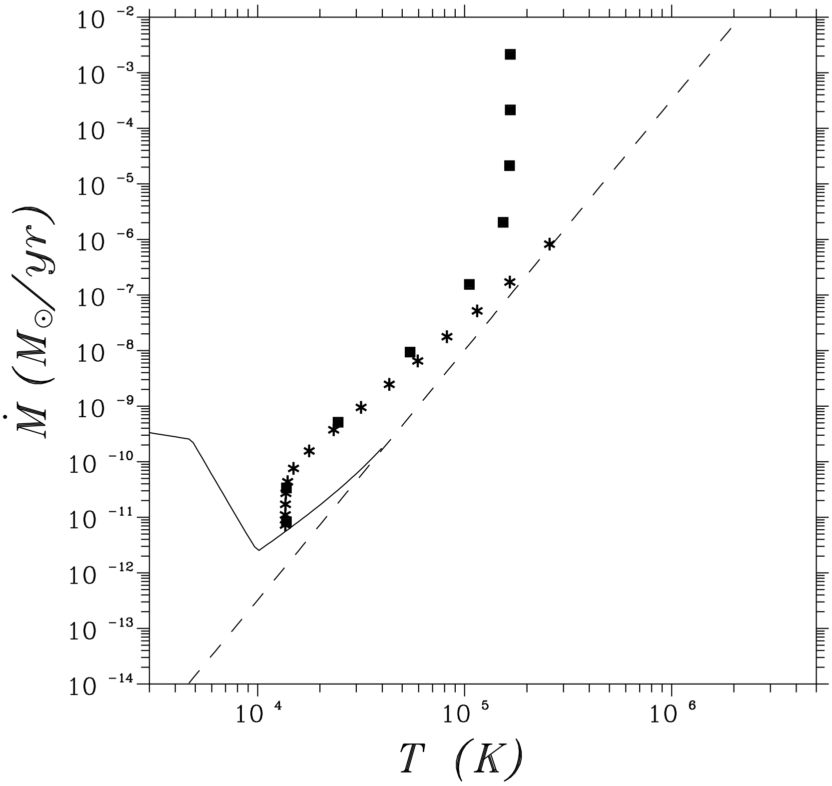

Figure 3a. Solution of (5.1) for an optically thick disk for $\alpha = 1$ and $r = A/5$ (asterisks). Solutions of (5.2) taking into account convection are labeled by squares. The dashed line is a lower bound for the region where solutions of (3.1) exist, and the solid line separates the regions of optically thin and optically thick disks.

with the same boundary conditions as for (5.1). Equation (5.2) does not admit a simple analytical solution, and we solved this equation numerically using the method of establishment. The solution is denoted by the squares in Figs. 3a – 3c. We can see that convection plays a significant role only when $\alpha \simeq 1$.

Summarizing, we can assert that, in the optically thick case with small $\dot{M}$, the disk displays the constant temperature $T = T_* = 13600°$ K, while the temperature increases as $T \propto \dot{M}^{1/3}$ at larger values of $\dot{M}$. Thus, for realistic parameters of the accretion disks in close binaries, $\dot{M} \simeq 10^{-12} \div 10^{-7} M_\odot/year$ and $\alpha \simeq 10^{-1} \div 10^{-2}$, the gas temperature in the outer parts of the disk ($r \simeq A/5 \div A/10$) is from $\sim 10^4$ K to $\sim 5 \times 10^5$ K.

Solving (5.1) for various r, we can also calculate the dependences $T(r)$ and $\rho(r)$. The calculations indicate that $T \propto r^{-0.8}$ and $\rho \propto r^{-1.8}$, which is consistent with the dependence $T \propto r^{-3/4}$ obtained by Shakura and Sunyaev (1973).

6. Opically thin disks

In this case, the temperature of the disk is specified by the balance between radiative heating $\Gamma(T, T_{wd})$ and viscous heating (2.5), on the one hand, and radiative cooling $\Lambda(T)$, on the other. The heat-balance equation (2.1a) can be written

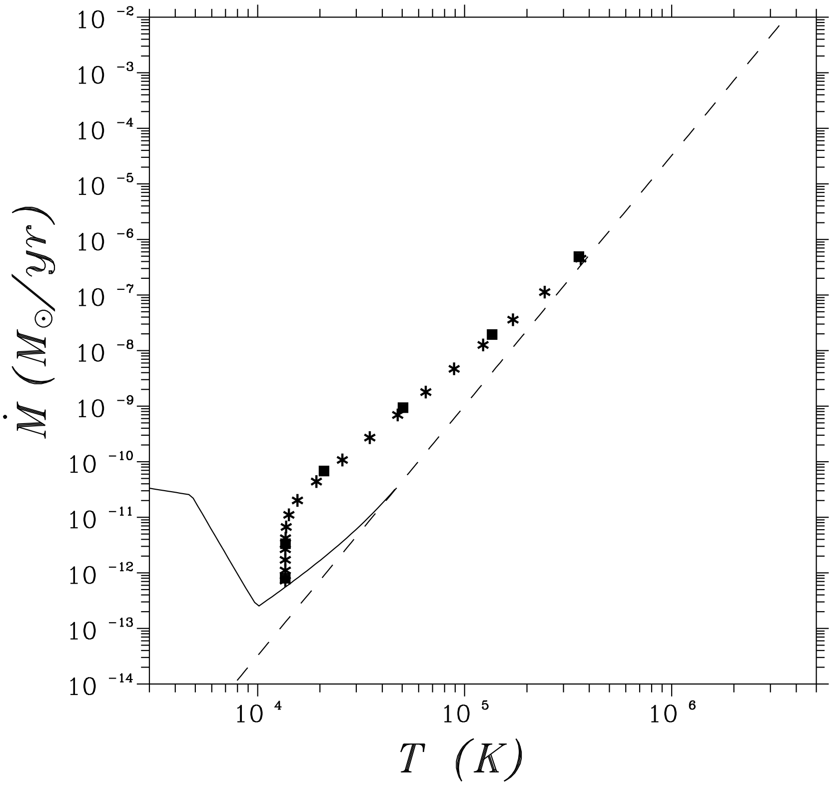

Figure 3b. Same as Fig. 3a for $\alpha = 0.1$.

$$\rho\alpha c_s^2\Omega_K + \rho^2 \cdot m_p^2 \cdot (\Gamma(T,T_{wd}) - \Lambda(T)) = 0\,,$$

which can be reduced to the quadratic equation in ρ

$$\alpha \cdot (\rho\mathcal{R}T + {}^1\!/_3 aT^4) \cdot \Omega_K + \rho^2 \cdot m_p^{-2} \cdot (\Gamma(T,T_{wd}) - \Lambda(T)) = 0\,.$$

The solution of this equation for specified r and α yields the dependence $\rho(T)$, and thus $T(\dot{M})$. Formally, this solution can also exist in optically thick regions; however, these points were rejected by virtue of the adopted assumptions.

It is shown in Section 4 that disks in which gas pressure dominates are primarily optically thick, and solutions that correspond to optically thin disks are possible only for small $\dot{M}$. Disks in which radiation pressure dominates are primarily optically thin. The domination of radiation pressure is possible only in the inner parts of the disk; therefore, we will adopt $r = A/20$ for the further analysis. For the typical dwarf nova IP Peg, this corresponds to five radii of the accretor (white dwarf).

Figure 4 presents the results of our calculations; the asterisks denote the $T(\dot{M})$ dependences for $\alpha = 1$, $\alpha = 0.1$, $\alpha = 10^{-2}$, $\alpha = 10^{-3}$ (top to bottom), and $r = A/20$. The disks obtained in these solutions are geometrically thick, $H \simeq r$. Note that the initial assumptions of the model restrict its applicability: it is suitable only for geometrically thin disks, and the solutions for geometrically thick disks are purely formal.

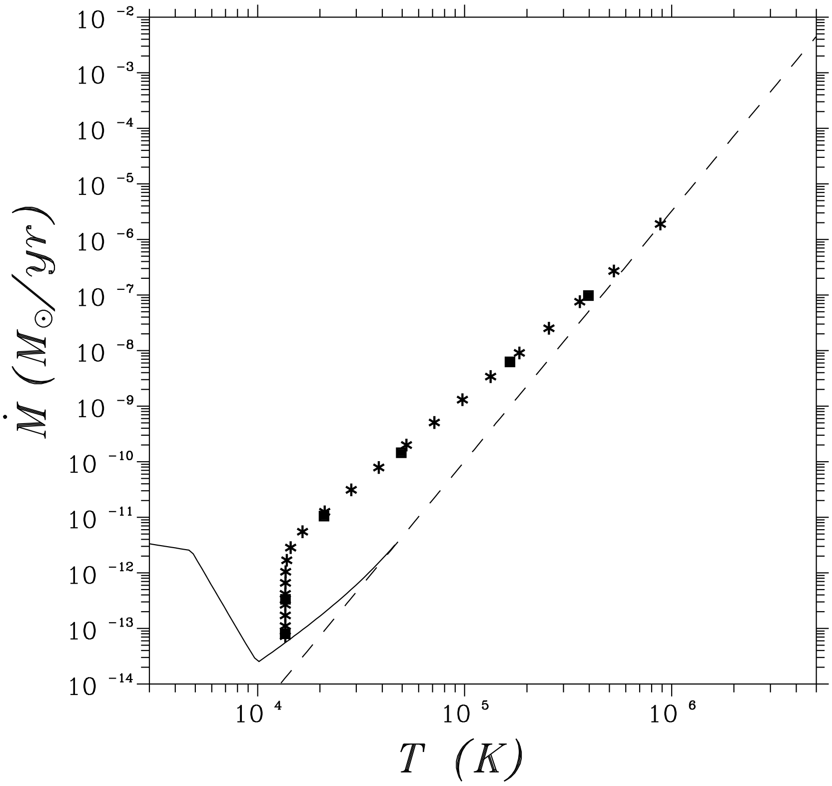

Figure 3c. Same as Fig. 3a for $\alpha = 0.01$.

7. Forms of the governing equations suited for the study of semidetached binaries

7.1. *Thin disks*

Let us consider first the cold disks. A thin disk may be described in terms of the surface density (i.e., the mass per unit surface area)

$$\Sigma = 2\int_0^H \rho \ \mathrm{d}\, z\,,$$

where H is the half-thickness of the disk. Time variations of the surface density Σ define the dynamical behaviour of the accretion disk. Dissipation processes in a disk result in matter motion in the radial direction. This motion will hereafter be characterized by a radial gas velocity v_r. The equations describing the behaviour of a thin disk can be deduced from a complete system of three-dimensional gas dynamical equations upon integrating over the z-direction and assuming that this is a Keplerian disk (i.e., $v_\phi = v_K = r\Omega_K = \sqrt{GM_2/r}$) and v_r is independent of z. It follows from the law of conservation of matter that at radius r

$$r\frac{\partial\Sigma}{\partial t} + \frac{\partial}{\partial r} r v_r \Sigma = 0\,. \tag{7.1}$$

From the law of conservation of angular momentum we have:

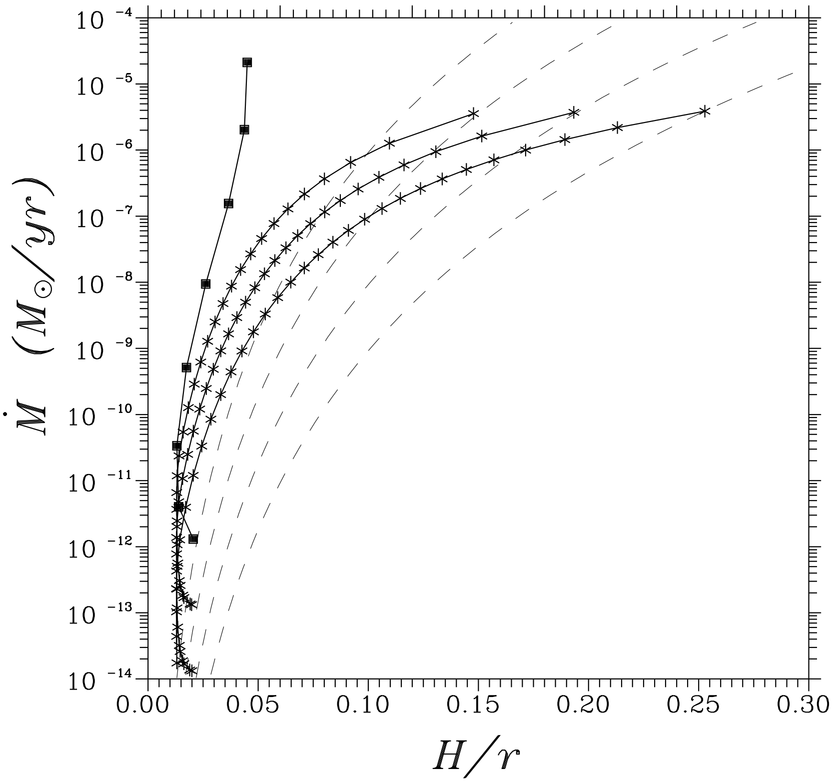

Figure 3d. A disk with dominant gas pressure. The solid lines indicate possible states of the disk in the $H/r - \dot{M}$ plane for $r = A/5$. Solutions of (5.1) taking into account radiative heat conduction and viscous heating are shown by the lines with asterisks for $\alpha = 0.1$, $\alpha = 10^{-2}$, and $\alpha = 10^{-3}$ (top to bottom). Solutions of (5.2) taking into account radiative heat conduction, convection, and viscous heating are shown by the lines with squares for $\alpha = 1$. The dashed lines bound from below regions in which the solution of (3.1) can exist for $\alpha = 1$, $\alpha = 0.1$, $\alpha = 10^{-2}$, and $\alpha = 10^{-3}$ (top to bottom).

$$r\frac{\partial}{\partial t}\Sigma r^2\Omega + \frac{\partial}{\partial r} r v_r \Sigma r^2 \Omega = \mathcal{G}\,, \tag{7.2}$$

where

$$\mathcal{G} = \frac{1}{2\pi}\frac{\partial G}{\partial r}\,,$$

and G is the torque of the viscous force at radius r:

$$G = 2\pi r \cdot \nu \Sigma r \cdot \frac{\partial \Omega}{\partial r} \cdot r\,.$$

Here ν is the kinematic viscosity coefficient, and $r \cdot \partial\Omega/\partial r$ is the rate of shearing. Combining (7.1) and (7.2) we will obtain the equations describing the time variations of the surface density Σ and radial velocity v_r for a Keplerian disk:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left(\sqrt{r}\frac{\partial}{\partial r}\left(\nu\Sigma\sqrt{r}\right)\right)\,, \tag{7.3}$$

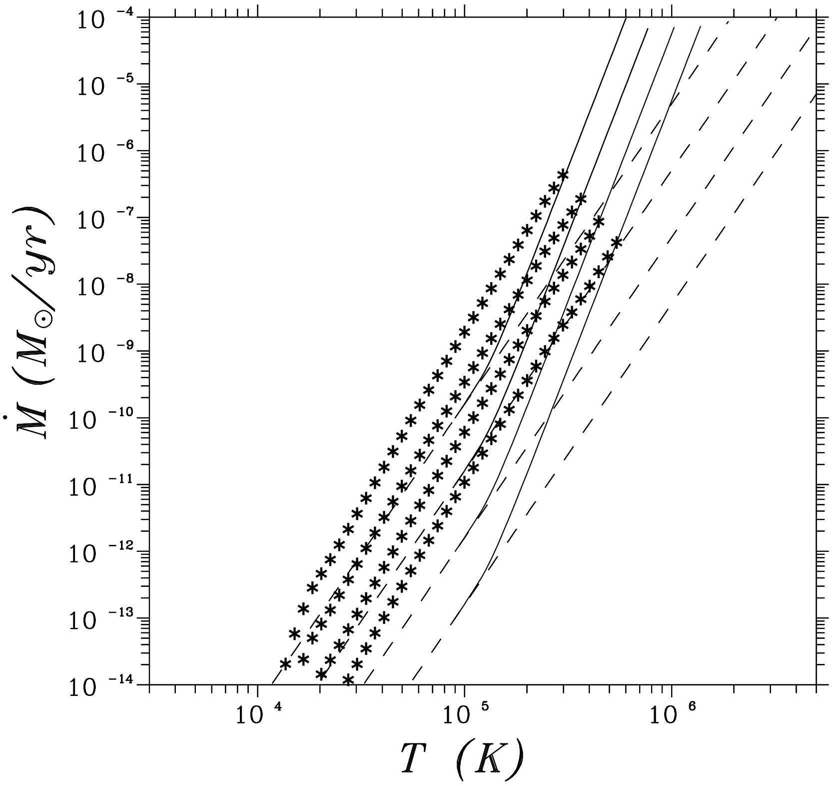

Figure 4. Solutions for an optically thin disk for $\alpha = 1$, $\alpha = 0.1$, $\alpha = 10^{-2}$, $\alpha = 10^{-3}$ (top to bottom) and $r = A/20$ (asterisks). The dashed lines bound frombelow the domain inwhich there exists a solution of (3.1); the solid lines separate the regions for optically thin and optically thick disks.

$$v_r = -\frac{3}{\Sigma\sqrt{r}}\frac{\partial}{\partial r}\nu\Sigma\sqrt{r}\,. \tag{7.4}$$

An important property of Eq. (7.3) is that this is a diffusion-type equation. It implies that the surface density of a disk at a given radius r can vary only in terms of the viscous time scale

$$t_\nu(r) \sim \frac{r^2}{\nu}\,, \tag{7.5}$$

or, in other words, the radial velocity of gas motion through a disk is proportional to $\sim r/t_\nu \sim \nu/r$. From the physical point of view, the diffusive behaviour of radial mass transfer in a disk is caused by an excess of angular momentum of the gas. Accretion cannot occur until the gas gets rid of excessive angular momentum.

In astrophysical disks gas is subject not only to gravitation and centrifugal force but also some additional forces, for example, the pressure gradient. This implies that these disks rotate owing to a law other than the Keplerian law. In this case the dynamics of the gas of the disk is described by the equation of momentum conservation in the radial direction and the equation of hydrostatic equilibrium in the vertical direction. Assuming axial symmetry and a polytropic equation of state for the gas of the disk, one can write the relevant system of equations as follows:

$$\begin{cases} \dfrac{\partial v_r}{\partial t} + v_r \dfrac{\partial v_r}{\partial r} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial r} = -\dfrac{\partial \Phi}{\partial r} + \dfrac{v_\phi^2}{r} \\ \dfrac{1}{\rho}\dfrac{\partial p}{\partial z} = -\dfrac{\partial \Phi}{\partial z} \\ p = K\rho^{1+1/n}\,. \end{cases} \tag{7.6}$$

To describe thin disks, a standard system of "planar" equations averaged over z is most frequently used:

$$\begin{cases} \dfrac{\partial v_r}{\partial t} + v_r \dfrac{\partial v_r}{\partial r} + \dfrac{1}{\Sigma}\dfrac{\partial \Pi}{\partial r} = -\dfrac{\partial \Phi_0}{\partial r} + \dfrac{v_\phi^2}{r} \\ \Pi = K_1 \Sigma^{1+1/n_1}\,, \end{cases} \tag{7.7}$$

where Σ is as usual the surface density, and

$$\Pi = 2\int\limits_0^H p \;\mathrm{d}\,z$$

is the vertically integrated (i.e. "planar") pressure. In system (7.7) Φ_0 is the first term of the expansion of the potential Φ with respect to the variable z in the vicinity of the equatorial plane (the prime designates differentiation with respect to z):

$$\Phi = \Phi_0(r) + {}^1/_2 \cdot \Phi_0''(r)\cdot z^2 + \cdots\,,$$

and n_1 is the 'planar' polytropic index. However, equations (7.6) and (7.7) are generally not equivalent (Popham & Narayan, 1991; Fridman & Khoruzhii, 1994). Indeed, it follows from the second equation of system (7.6) that the function

$$\chi = \Phi + \int \frac{\mathrm{d}\,p}{\rho}$$

depends only on r and, hence, the density ρ can be written as

$$\rho = \left[\frac{\chi(r) - \Phi_0(r) - {}^1/_2\cdot\Phi_0''(r)\cdot z^2}{K(n+1)}\right]^n\,.$$

Integrating this expression for density over z from $-H$ to H and taking into account that the half-thickness of the disk can be determined here as $H = \sqrt{2(\chi-\Phi_0)/\Phi_0''}$, we obtain the relationship between χ and Σ:

$$\chi(r) = \Phi_0(r) + C(r)\cdot\Sigma^{\frac{2}{2n+1}}\,.$$

To obtain the "planar" equation of momentum conservation in the radial direction with correct averaging, we should differentiate $\chi(r)$ with respect to r. As a result we obtain:

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{2}{2n+1}C(r)\Sigma^{-\frac{2n-1}{2n+1}} + \frac{dC}{dr}\cdot\Sigma^{\frac{2}{2n+1}} = -\frac{\partial\Phi_0}{\partial r} + \frac{v_\phi^2}{r}\,. \tag{7.8}$$

This equation differs from the appropriate equation of the system (7.7) which can be written as

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + K_1 \frac{n_1+1}{n_1} \Sigma^{-\frac{n_1-1}{n_1}} = -\frac{\partial \Phi_0}{\partial r} + \frac{v_\phi^2}{r} \,. \qquad (7.9)$$

It is obvious that equation (7.8) differs from the standard equation (7.9) not only by the presence of an additional term containing dC/dr, but also different exponents at Σ in the term describing the pressure gradient when identical polytropic indexes $n_1 = n$ are used for the complete and "planar" model. These differences disappear only in the isothermal case† corresponding to $n \to \infty$, because in this case $dC/dr \to 0$. For the axisymmetrical polytropic case the disk can be described within the framework of the "planar" model by modified the equation of momentum in the radial direction (7.8).

Analysis of the modified "planar" equations for the non-axisymmetrical polytropic case (Fridman & Khoruzhii, 1994) is very sophisticated; therefore to describe the accretion disks in an external gravitational field, it is easier to use the complete three-dimensional equations. As to the more general case when the matter of the disk is not described by the barotropic equation of state and can be written as $p = p(\rho, \varepsilon)$,‡ it is still unclear whether the closed system of "planar" equations can be deduced correctly or not. Note that the three-dimensional approach becomes even more important because the averaged "planar" approach cannot be applied to thick disks.

7.2. *Thick disks*

To analyse flow structure in binary we will use the system of Euler equations that describes the flow of a compressed non-viscous gas. In the general case, to describe gas flow in real binaries, one should take into account many physical processes that are not included in the system of Euler equations. To take into account additional effects, one should add additional equations to the system of Euler equations, and/or add terms on the right-hand side of the original equations. In particular, this can be used for consideration of the evolution of the magnetic field and its influence on gas flow (equations of magnetic hydrodynamics), for the analysis of the radiation field and its impact on the structure of gas flow (radiation gas dynamics), and also for the account of gravitation and self-gravitation.

We use the Euler equations to describe flow structure in binary systems. The main reason of this choice is that studying all the variety of physical processes occurring in real systems is a multistage process, and as the first step we should study the basic ("managing") processes. In our case – that is the study of gas flow in semidetached binaries – apparently, Euler equations are sufficient to describe the managing processes (see, e.g., Boyarchuk et al., 2002). We should also note that the present level of computer facilities corresponds to this step of the study, and, although it is theoretically clear how we should improve the numerical models, but for the time being we cannot use this way because it is too expensive.

The Euler gas dynamical equations can be written in non-divergence form as:

† We should note that similar considerations for the case of a self-gravitated disk proves that the standard "planar" equations are correct only for $n = 1$.

‡ For example, the equation of state for the ideal gas $p = (\gamma - 1)\rho\varepsilon$.

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \mathbf{v}\,\mathrm{grad}\,\rho + \rho\,\mathrm{div}\,\mathbf{v} = 0 \\[2ex] \dfrac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \dfrac{1}{\rho}\,\mathrm{grad}\,p = -\,\mathrm{grad}\,\Phi \\[2ex] \dfrac{\partial \varepsilon}{\partial t} + \mathbf{v}\,\mathrm{grad}\,\varepsilon + \dfrac{p}{\rho}\,\mathrm{div}\,\mathbf{v} = 0\,. \end{cases} \tag{7.10}$$

In (7.10) the term "$-\,\mathrm{grad}\,\Phi$" describing gravitation in the binary star is included (here Φ is the gravitational potential). To close a system of equations, one should add the equation of state $p = p(\rho, \varepsilon)$ (for example, the equation of an ideal gas $p = (\gamma - 1)\varepsilon\rho$) and the equation for the determination of the gravitational potential Φ (in the case of self-gravitation it should be determined through Poisson's equation $\Delta\Phi = 4\pi G\rho$, while if self-gravitation is negligible we can simply write $\Phi = -GM/r$).

In divergence form, the Euler gas dynamical equations can be written as:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \mathrm{div}(\rho\mathbf{v}) = 0 \\[2ex] \dfrac{\partial \rho\mathbf{v}}{\partial t} + \mathrm{div}\,\mathcal{T} = \mathbf{F} \\[2ex] \dfrac{\partial \rho E}{\partial t} + \mathrm{div}(\rho h\mathbf{v}) = \mathbf{F}\cdot\mathbf{v}\,, \end{cases} \tag{7.11}$$

where

$$\mathcal{T} = \begin{pmatrix} \rho u^2 + p & \rho uv & \rho uw \\ \rho uv & \rho v^2 + p & \rho vw \\ \rho uw & \rho vw & \rho w^2 + p \end{pmatrix}.$$

In equations (7.10) and (7.11), $\mathbf{v}$ is the velocity vector, with components (u, v, w); $E = \varepsilon + \mathbf{v}^2/2$ is the full specific energy (per mass unit); ε is the internal specific energy (per mass unit); $h = \varepsilon + p/\rho + \mathbf{v}^2/2$ is the full specific enthalpy (per mass unit); and $\mathbf{F}$ is the specific external force (per volume unit).

The divergence equations (7.11) can be also written in the integral form:

$$\begin{cases} \dfrac{\partial}{\partial t}\displaystyle\int_V \,\mathrm{d}\,\mathbf{r} = 0 \\[2ex] \dfrac{\partial}{\partial t}\displaystyle\int_V \rho\;\mathrm{d}\,\mathbf{r} + \int_\Sigma \rho(\mathbf{v}\cdot\mathbf{n})\;\mathrm{d}\,s = 0 \\[2ex] \dfrac{\partial}{\partial t}\displaystyle\int_V \rho\mathbf{v}\;\mathrm{d}\,\mathbf{r} + \int_\Sigma \Big(\rho\mathbf{v}(\mathbf{v}\cdot\mathbf{n}) + p\mathbf{n}\Big)\;\mathrm{d}\,s = \int_V \mathbf{F}\;\mathrm{d}\,\mathbf{r} \\[2ex] \dfrac{\partial}{\partial t}\displaystyle\int_V \rho E\;\mathrm{d}\,\mathbf{r} + \int_\Sigma \rho h(\mathbf{v}\cdot\mathbf{n})\;\mathrm{d}\,s = \int_V (\mathbf{F}\cdot\mathbf{v})\;\mathrm{d}\,\mathbf{r}\,. \end{cases} \tag{7.12}$$

In this form equations (7.12) do not require the assumptions of differentiability (and even continuity) of macroscopic functions and hence they can be used to describe gas dynamical flow with shock waves and contact discontinuities. Note that the first equation of (7.12) implies that all relations are written for a unit volume that has a constant position in space. This corresponds to the case of Euler variables when the spatial coordinates do not change in time $\partial\mathbf{r}/\partial t \equiv 0$ and, hence, differentiation with respect to t is made at constant $\mathbf{r}$.

Alternatively, the original system of equations can be written using Lagrange variables, i.e., when the relations are written for a "gas element" moving together with matter. In other words, these variables can be determined as the case when spatial coordinates change in time along with the motion of matter $D\mathbf{r}/Dt = \mathbf{v}$. Usually "gas elements" are marked by their position $\mathbf{r}_0$ at $t = 0$, the mark of a "gas element" being constant. The operator D/Dt is referred to as the Lagrangian time derivative, or in other words is a partial derivative with respect to t at constant $\mathbf{r}_0$. The conservation laws in integral form can be obtained from (7.12) using the following relation:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + (\mathbf{v}\cdot\nabla)\,,$$

$$\frac{D}{Dt}\Delta = \Delta\,\mathrm{div}\,\mathbf{v}\,,$$

where Δ is the Jacobian determinant of the transformation from $\mathbf{r}_0$ to $\mathbf{r}$:

$$\Delta = \left|\frac{\partial\mathbf{r}}{\partial\mathbf{r}_0}\right|\,.$$

For the sake of simplicity we consider the case when external forces are absent. After simple transformations we obtain the equations of gas dynamics with Lagrange variables in the integral form:

$$\left\{\begin{aligned}
&\frac{D}{Dt}\int\limits_V \rho\ \mathrm{d}\,\mathbf{r} = \frac{D}{Dt}\int\limits_M \mathrm{d}\,m = 0\\
&\frac{D}{Dt}\int\limits_V \mathrm{d}\,\mathbf{r} = \frac{D}{Dt}\int\limits_M \rho^{-1}\ \mathrm{d}\,m = \frac{D}{Dt}\int\limits_V \Delta\ \mathrm{d}\,\mathbf{r}_0 = \int\limits_M \rho^{-1}\,\mathrm{div}\,\mathbf{v}\ \mathrm{d}\,m\\
&\frac{D}{Dt}\int\limits_V \rho\mathbf{v}\ \mathrm{d}\,\mathbf{r} = \frac{D}{Dt}\int\limits_M \mathbf{v}\ \mathrm{d}\,m\\
&\qquad = \int\limits_V \Big(-\,\mathrm{div}\,\mathcal{T} + (\mathbf{v}\cdot\nabla)(\rho\mathbf{v}) + \rho\mathbf{v}\,\mathrm{div}\,\mathbf{v}\Big)\ \mathrm{d}\,\mathbf{r}\\
&\qquad = -\int\limits_V \mathrm{grad}\,p\ \mathrm{d}\,\mathbf{r} = -\int\limits_M \rho^{-1}\,\mathrm{grad}\,p\ \mathrm{d}\,m\\
&\frac{D}{Dt}\int\limits_V \rho E\ \mathrm{d}\,\mathbf{r} = \frac{D}{Dt}\int\limits_M E\ \mathrm{d}\,m = -\int\limits_M \rho^{-1}\,\mathrm{div}(p\mathbf{v})\ \mathrm{d}\,m\,.
\end{aligned}\right. \tag{7.13}$$

From the integral equations (7.13) we can deduce the differential equations of gas dynamics with Lagrange variables. These equations in divergence form can be written as:

$$\begin{cases} \dfrac{D}{Dt}\rho^{-1} - \rho^{-1}\operatorname{div}\mathbf{v} = 0 \\[2ex] \dfrac{D}{Dt}\mathbf{v} + \rho^{-1}\operatorname{grad} p = 0 \\[2ex] \dfrac{D}{Dt}E + \rho^{-1}\operatorname{div}(p\mathbf{v}) = 0\,. \end{cases} \tag{7.14}$$

To close the system we should add the motion equation for the Euler variable $\mathbf{r}$

$$\frac{D\mathbf{r}}{Dt} = \mathbf{v}\,,$$

and, as usual, the equation of state. It should be noted that the energy equation in (7.14) is frequently written in the non-divergence form:

$$\frac{D\varepsilon}{Dt} + \frac{p}{\rho}\operatorname{div}\mathbf{v} = \frac{D\varepsilon}{Dt} + p\frac{D}{Dt}\rho^{-1} = 0\,. \tag{7.15}$$

For the majority of astrophysical applications gas flow is analysed using the Euler variables. However, in specific cases the equations with Lagrange variables (7.14) are also used, for example, by the SPH method.

Use of the Euler equations for the description of the gas flow in binary systems implies the absence of physical viscosity in the gas. Nevertheless, in the numerical solution of a system of equations the so-called numerical viscosity significantly affects the gas flow patterns. In particular, dissipation in the system causes the gas to heat. Therefore, to construct accurate gas dynamical models one should take into account radiation that results in cooling of the gas. Generally, radiation should be taken into account via including into the right-hand sides of the equations the terms describing energy radiation and absorption at every point of space. In a complete statement this problem is extremely difficult, because it is necessary to take into account the radiation coming to a point from all points of the domain.

However, we may simplify the model assuming that radiation is not absorbed in the system (the so-called optically thin case). This situation takes place for many astrophysical objects. In this case the sink term in the right-hand side of the energy equation can be represented by an expression causing the temperature to tend exponentially to a certain value T_0 in a time scale τ_{cool}:

$$\frac{\partial\rho E}{\partial t} + \operatorname{div}(\rho h\mathbf{v}) = \mathbf{F}\cdot\mathbf{v} - \rho R\frac{T - T_0}{\tau_{cool}}\,.$$

If the radiation time scale is smaller than the gas dynamical scale, the energy equation written with this sink term makes the system of gas dynamical equation a "stiff" one that can complicate its solution. To avoid solving this "stiff" system the isothermal gas dynamics equations could be used. The temperature for the optically thin case is practically constant, therefore the equation of state can be written as $p = R\rho T_0 \equiv c_T^2\rho$, where c_T is the isothermal sound velocity ($c_T = c_s/\sqrt{\gamma}$). Using this expression for p we can write the system of isothermal gas dynamical equations in the following form:

$$\begin{cases} \dfrac{\partial\rho}{\partial t} + \mathrm{div}(\rho\mathbf{v}) = 0 \\ \\ \dfrac{\partial\rho\mathbf{v}}{\partial t} + \mathrm{div}\,\mathcal{T} = \mathbf{F}\,, \end{cases}$$

where

$$\mathcal{T} = \begin{pmatrix} \rho u^2 + c_T^2\rho & \rho uv & \rho uw \\ \rho uv & \rho v^2 + c_T^2\rho & \rho vw \\ \rho uw & \rho vw & \rho w^2 + c_T^2\rho \end{pmatrix}.$$

In this system only the equations of mass and momentum conservation are presented. The energy equation is not included in the system because the temperature of the gas is *a priori* constant.

In practice, to describe the gas flow with radiative cooling neither the system with "stiff" sink term nor the system of isothermal gas dynamical equations are solved. Instead, the Euler gas dynamics equation and the equation of state of an ideal gas are solved, but the ratio of heat capacities γ is taken very close to 1 (for example, $\gamma = 1.01$). This case, according to the well-known formula from statistical physics $\gamma = (N+2)/N$, corresponds to a high number of inner degrees of freedom N of the gas molecules. Therefore, if the gas is heated (for example, due to shock waves or viscous dissipation), the heating will be spread around among so many degrees of freedom that it has little impact on the pressure and/or temperature. So this approach mimics a system with constant temperature, i.e. with effective radiative cooling.

8. Conclusions

Our analysis of the basic processes of heating and cooling in accretion disks in binaries has shown that, for realistic parameters of the accretion disks in close binary systems ($\dot{M} \simeq 10^{-12} \div 10^{-7} M_\odot/year$ and $\alpha \simeq 10^{-1} \div 10^{-2}$), the corresponding gas temperature in the outer parts of the disk is from $\sim 10^4$ K to $\sim 5\times10^5$ K. The forms of governing equations suited both for thin and thick accretion discs are discussed.

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