MHD TURBULENCE IN THE EARLY UNIVERSE AND PRIMORDIAL MAGNETIC FIELDS

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1-10 July, 2006

BG-URSI School and Workshop on Waves and Turbulence Phenomena in Cosmic Plasma.

Kiten, Bulgaria

Early (and not so early) universe is assumed to be smooth with low peculiar velocities and weak laminar flows. Vector perturbations are absent or negligible. Well agree with observations at large scales.

Can there be significant vorticity perturbations, turbulence?

Do we need them?

May explain existence of large scale magnetic fields and predict primordial gravitational waves.

CONTENT

1. Large scale magnetic fields and models of their creation. Mini-review.

2. Vorticity and turbulence in primeval plasma, possible mechanisms

3. Non-linear generation of vorticity at recombination epoch.

4. Large isocurvature fluctuations at neutrino decoupling, primordial turbulence, and gravitational waves.

Natural system of units.

$$\mathbf{c = h/2\pi = k = 1}$$

$$\mathbf{m_p = 10^{-24}\,g = 0.94\,GeV}$$
$$\mathbf{1^oK \approx 10^{-4}eV}$$
$$\mathbf{1/m_p = 2 \cdot 10^{-14}\,cm = 6.6 \cdot 10^{-25}\,sec}$$
$$\mathbf{1Gauss = 10^8\,cm^{-2}}$$

COSMIC MAGNETIC FIELDS

OBSERVATIONS:

Galactic magnetic fields,
B $\sim$ (a few) μGauss,
coherent at several kpc.
Intergalactic magnetic fields:
no strong evidence. Possibly

$$B_{int.gal} \sim 10^{-3} B_{gal},$$

scale: $\sim$ Mpc.
Much larger fields in filaments (?):
B $\sim$ μG, scale $\sim$ 10^2 kpc.

Adiabatic compression: $\mathbf{B \sim 1/l^2}$:

$$\mathbf{l_{gal}^{(in)}/l_{gal} \sim 10^2},$$
$$\mathbf{l_{ig}^{(in)}/l_{ig} \sim 3},$$

Expect $B_{gal} \sim 10^3 B_{ig}$, if common origin and no galactic dynamo amplification.

Possible galactic dynamo amplifies by (according to different sources):

$$\mathbf{10^{15\pm 5}}$$

(probably weaker than $\mathbf{10^{15}}$).
Still uncertain.

POSSIBLE SCENARIOS OF MAGNETIC FIELD GENERATION

1. "OLD UNIVERSE": Conventional astrophysical: ejecta from stars and field line reconnection.

2. "YOUNG UNIVERSE": Recombination and/or structure formation epochs, t=100000 years or later.

3."EARLY UNIVERSE": From inflation down to neutrino decoupling, t=1 sec or earlier.

STELLAR EJECTA

Total energy of magnetic field in a spiral galaxy:

$$\mathcal{E}_{gal}^{B} = \frac{4}{3}\pi R_{gal}^2 h_{gal} \rho_B \sim M_\odot$$

where

$$\rho_B = B^2/8\pi \sim \rho_{CMB} \approx 5 \cdot 10^{-34} \, g/cm^3$$

For comparison: $\rho_{tot} \approx 10^{-29} g/cm^3$ **and** $\rho_m \approx 0.3 \cdot 10^{-29} g/cm^3$.
Dark energy: $\rho_m \approx 0.7 \cdot 10^{-29} g/cm^3$.

Neutron stars:

$$\mathcal{E}^{\mathbf{B}}_{\mathbf{ns}} \approx \mathbf{10^{-11} M_{\odot}} \left(\frac{\mathbf{B}}{\mathbf{10^{13} G}}\right)^{\mathbf{2}} \left(\frac{\mathbf{R_{ns}}}{\mathbf{10^{6} cm}}\right)^{\mathbf{3}}$$

White dwarfs:

$$\mathcal{E}^{\mathbf{B}}_{\mathbf{wd}} \approx \mathbf{10^{-10} M_{\odot}} \left(\frac{\mathbf{B}}{\mathbf{10^{9} G}}\right)^{\mathbf{2}} \left(\frac{\mathbf{R_{wd}}}{\mathbf{10^{9} cm}}\right)^{\mathbf{3}}$$

Sun (or normal stars:

$$\mathcal{E}_{\odot}^{\mathrm{B}} \approx 10^{-17}\mathrm{M}_{\odot}\left(\frac{\mathrm{B}}{10^{3}\mathrm{G}}\right)^{2}\left(\frac{\mathrm{R}_{\odot}}{10^{11}\mathrm{cm}}\right)^{3}$$

Power loss in reconnection should be large, $\sim (\mathrm{R_{star}}/\mathrm{R_{gal}})^{3}$.

More stars than exist in galaxy are needed.

INFLATIONARY GENERATION

Production of photons by external gravitational field and inflating their wave length by $> \exp(60)$ creates classical (quasi)stationary electromagnetic field.

INFLATION: period of exponential expansion, $a \sim \exp[Ht]$, - experimental fact.. Mostly, $a \sim t^{1/2}$ or $t^{2/3}$; now again we expand exponentially.

For production of particles by gravitational field breaking of conformal invariance is necessary.

In conformally invariant case gravity can be switched off by rescaling. FRW metrics are conformally flat:

$$\mathbf{ds^2 = a^2(\eta, r)(d\eta^2 - dr^2)}$$

and can be formally transformed into flat space-time by $\mathbf{g_{\mu\nu} \rightarrow g_{\mu\nu}/a^2}$.

Equation of motion of massless fields in gravitational field can take the free form if the fields are rescaled as $\Phi \to [a(t,x)]^k \Phi$ (the exponent k depends upon the spin of Φ):

$$(\partial_\eta^2 - \Delta)\Phi = 0$$

Particles are not produced (Bogolyubov coefficients are trivial).

No mass - no scale!

Photons and massless fermions are conformally invariant.

But, surprise: scalars and gravitons are not.

Quanta of scalar field are known to be created at inflation and generate density perturbations with observed almost flat spectrum. We are here because of that.

However, Classical Electrodynamics is conformally invariant.

Small masses of fermions can be neglected, or they even vanished prior to Higgs condensation.

The invariance may be broken either by new hypothetical interaction or by quantum effects.

1. New coupling of e.m. field to gravity:

$$\mathcal{L} = \mathbf{C_1 R A}_\mu \mathbf{A}^\mu + \mathbf{C_2 R}_{\mu\nu} \mathbf{A}^\mu \mathbf{A}^\nu + \mathbf{C_3 R}_{\mu\nu\alpha\beta} \mathbf{F}^{\mu\nu} \mathbf{F}^{\alpha\beta} + ...$$

Gauge invariance is broken by the first term. The others may be generated by radiative corrections. However, radiative corrections should normally give too small coefficients.

2. New interaction with dilaton, θ:

$$\mathcal{L} = -(1/4)\, e^{\theta}\, F_{\mu\nu} F^{\mu\nu}$$

Renormalizability may be broken.
Should we care?
Probably should.

3. Quantum conformal anomaly:

$$\mathbf{T}^{\mu}_{\mu} = \frac{\alpha}{\pi}\left(\frac{\mathbf{11N}}{\mathbf{3}} - \frac{\mathbf{2N_f}}{\mathbf{3}}\right)\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}$$

Classically:

$$\mathbf{T}_{\mu\nu} = \mathbf{F}_{\mu\alpha}\mathbf{F}^{\alpha}_{\nu} - \frac{\mathbf{1}}{\mathbf{4}}\mathbf{g}_{\mu\nu}\mathbf{F}^{\mathbf{2}}$$

and $\mathbf{T}^{\mu}_{\mu} = \mathbf{0}$**.**

Quantum effects break classical conservation laws or symmetries. Examples: chiral and conformal anomalies.

In all the cases galactic dynamo is needed, about 10^{10} or more.

Wave lengths are OK: $\lambda > e^{60}\lambda_{in}$, but amplitudes are not large enough.

On the other hand, the mechanisms are quite exotic and demands new physics, except possibly for quantum anomaly.

PERTURBATIONS PRODUCING VORTICITY and electric currents

1. During (re/pre)-heating at the end of inflation. **Explosion of vacuum-like state governed by inflaton: "let there be light". Particle production is a chaotic process and chaotic inhomogeneities could be created. Scales are very short.**

2. First order phase transitions.

Breaking GUT symmetry, $T \sim 10^{16}$ GeV - may never existed.
Electroweak p.t. $T \sim$ TeV - II order.
QCD p.t. $T \sim 100$ MeV - most probably 2nd order(?)
At high T (in the early universe) gauge symmetries are restored, while now they are broken. Similar to symmetry restoration in ferromagnet at high T.

If 1st order p.t. boiling primeval plasma, bubbles of one phase inside another.
Large perturbations and magnetic fields, but very short wave lengths.
Horizon: $\mathbf{t/sec \sim 1/(T/MeV)^2}$.
E.g. $\mathbf{t_{EW} \sim 10^{-12}}$ **sec;**
red-shift $\mathbf{z \sim T_{EW}/T_{CMB} \approx 10^{16}}$
$\boldsymbol{\lambda_{now} \sim 10^4}$ **sec.**
Killed by diffusion, dissipation,...

3. Spatial fluctuations of leptonic charge by neutrino oscillations. **Took place at neutrino decoupling. Scale about 100 years.**

If $T \sim 1$ MeV, then $t \sim 1$ sec and this epoch has red-shift $z \approx 10^{10}$, i.e. the horizon is about 100 years in the present day scale.

4. Hydrogen recombination epoch.

Rather large scales but second order effect in density perturbations.

Could dissipation effects enhance vorticity generation? Not yet, because velocities are still too small.

Essential parameters.

$$\mathbf{H^{-1} = \frac{27\ kpc}{T_{\mathrm{eV}}^{3/2}\,[T_{eV} + 0.76]^{1/2}}},$$

where $\mathbf{T_{eV} = (T/1\ eV)}$, today it is:
$\mathbf{110\,T_{\mathrm{eV}}^{-1/2}[T_{\mathrm{eV}} + 0.76]^{-1/2}}$ Mpc.
Hydrogen recombination at
$\mathbf{T \approx 3000\,K^{o} \approx 0.25\ eV}$,
number of free electrons drops by $\mathbf{10^{5}}$.

Photon mean free path:

$$\nu = l_\gamma = \frac{1}{\sigma_T n_e X_e} \approx \frac{30\ \text{pc}}{X_e(T) T_{eV}^3},$$

$$\sigma_T = 8\pi\alpha^2/3m_e^2 = 6.65 \times 10^{-25}\, \text{cm}^2$$

is the Thomson cross-section, X_e is fraction of free electrons, and

$$n_e = \beta n_\gamma = 6 \cdot 10^{-10}$$

5. Onset of structure formation. Rising perturbations, they are of potential type but: vorticity perturbations by dissipation.
Smaller number of charge carriers because of recombination.
Maybe re-ionization period at z $\sim$ 10 is favorable?

NECESSITY OF VORTICITY
and basic equations

Magnetic hydrodynamics:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\kappa} \nabla \times \mathbf{J}$$

valid for large conductivity κ.
Since $\mathbf{J} = en\mathbf{v}$ vortex motion,

$$\nabla \times \mathbf{v} \neq 0$$

is necessary.

In fact: $\mathbf{J} \neq \kappa\mathbf{E}$,
because the current is induced by external force, by light pressure.
Equations to be used:

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = \mathbf{0},$$
$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -4\pi\mathbf{J}$$

Thus $\mathbf{B} \sim \nabla \times \mathbf{J}$. (It follows from parity conservation.)

Comment about current.

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -4\pi \mathbf{J}$$

Here $\mathbf{J} = \mathbf{J}_{\mathbf{external}} + \kappa \mathbf{E}$. This leads to vanishing of E for large conductivity.

Vorticity.

Hydrodynamic approximation:

$$\partial_t \mathbf{v} + (\mathbf{v}\,\nabla)\,\mathbf{v} - \nu \Delta \mathbf{v} = -\frac{\nabla p}{\rho}$$

where $\nu \sim \mathrm{l}_{\mathbf{free}}$ is viscosity and approximately homogeneous liquid is assumed. (More accurately, Navier-Stokes equation should be used.)

Vorticity, $\mathbf{\Omega} = \nabla \times \mathbf{v}$, satisfies:

$$\partial_t \mathbf{\Omega} - \nu \Delta \mathbf{\Omega} = S \equiv -\nabla \times \left(\frac{\nabla p}{\rho} \right)$$

If $p = w\rho$ with $w = w(t)$, then

$$\mathbf{S = 0}.$$

Maybe not true in multicomponent plasma.
What if the medium is inhomogeneous, e.g. $\nu = \nu(\mathbf{x})$? An additional term appears:

$$\epsilon_{\mathbf{ijk}}(\partial_{\mathbf{j}}\nu)\mathbf{\Delta V_k},$$

source of vorticity but also in the second order.

Local thermal equilibrium:

$$\mathbf{f} = \exp\left[-\frac{\mathbf{E}}{\mathbf{T}(\mathbf{x})} + \xi(\mathbf{x})\right]$$

Vorticity source:

$$\mathbf{S_k} \equiv -\epsilon_{\mathbf{ijk}}\,\partial_{\mathbf{j}}\left(\frac{\partial_{\mathbf{i}}\mathbf{p}}{\rho}\right) = \epsilon_{\mathbf{ijk}}\,\frac{\partial_{\mathbf{i}}\rho_\gamma}{\mathbf{3}\rho_{\rm tot}}\,\frac{\partial_{\mathbf{j}}\beta}{\beta}\,\frac{\rho_{\mathbf{b}}}{\rho_{\rm tot}}$$

where $\xi(\mathbf{x}) = \ln\beta(\mathbf{x}) + \mathrm{const}$, $\beta = \mathbf{n_b}/\mathbf{n}_\gamma$, and $\rho_{\mathbf{b}}$ is baryonic energy density.

Vectors $\nabla\rho_\gamma$ and $\nabla\beta$ should not be collinear, i.e. photons and baryons must have different spatial distributions - possible at acoustic oscillation epoch.

Non-stationarity effects.

Kinetic equation:

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - H\,\mathbf{p}\frac{\partial}{\partial \mathbf{p}} - \mathbf{F}\,\frac{\partial}{\partial \mathbf{p}}\right) f_\gamma(t,\mathbf{x},E,\mathbf{p}) = I_{\rm coll}\left[f_a, f_b, ...\right],$$

Perturbative solution

$$\mathbf{f} \approx \mathbf{f_0} + \mathbf{f_1}$$

where

$$\mathbf{f_0} = \exp\left[-\frac{\mathbf{E}}{\mathbf{T}(\mathbf{x},\mathbf{t})} + \xi(\mathbf{x},\mathbf{t})\right]$$

If collision integral is taken
in effective time approximation:

$$\mathbf{I}_{\rm coll} = -\mathbf{\Gamma}(\mathbf{f} - \mathbf{f_{eq}})$$

then:

$$\left(\mathbf{K} + \mathbf{\Gamma}\right)\mathbf{f_1} = -\mathbf{K}\mathbf{f_0}$$

where $K = \partial_t + (\mathbf{V}\,\nabla)$ **and:**

$$\mathbf{f_1}\left(\mathbf{t}, \mathbf{x}, \mathbf{E}, \mathbf{V}\right) = \int_{\mathbf{0}}^{\mathbf{t}} \mathbf{d}\tau_{\mathbf{1}} \mathbf{K}\mathbf{f_0}\left(\mathbf{t} - \tau_{\mathbf{1}}, \mathbf{x} - \mathbf{V}\tau_{\mathbf{1}}\right) \mathbf{e}^{-\int_{\mathbf{0}}^{\tau_{\mathbf{1}}} \mathbf{d}\tau_{\mathbf{2}} \mathbf{\Gamma}(\mathbf{t} - \tau_{\mathbf{2}}, \mathbf{x} - \mathbf{V}\tau_{\mathbf{2}})}$$

Macroscopic velocity:

$$v_j(t,\mathbf{x}) = \frac{\int d^3p V_j f_1(t,\mathbf{x},E,\mathbf{V})}{\int d^3p f_0(t,\mathbf{x},E)}$$

and vorticity, $\Omega_i = \epsilon_{ijk}\partial_j v_k$**:**

$$\mathbf{\Omega_i \approx \frac{6\epsilon_{ijn}}{\Gamma^2}\left(\frac{\partial_j T}{T}\right)\left(\frac{\partial_n \partial_t T}{T}\right)}$$

$\mathbf{\partial_t T}$ **can be taken either from diffusion equation** $\mathbf{\dot{T} = D\Delta T,}$ **or from cosmological expansion,** $\mathbf{\dot{T} = -HT,}$ **depending upon wave length.**

Non-vortical motion,

possible in first order in $\delta\rho/\rho$.

Solve hydrodynamic equation:

$$\partial_t \mathbf{v} + (\mathbf{v}\,\nabla)\,\mathbf{v} - \nu\Delta\mathbf{v} = -\frac{\nabla p}{\rho}$$

in the limit of small v

$$\mathbf{v_k} = -\frac{i\mathbf{k}}{3k^2\nu}\,\delta_k\,\left[1 - \exp(-\nu k^2 t)\right]$$

and find Reynolds number.

Reynolds number at eV temperatures:

$$\mathbf{(Re)_k \approx 4 \left(\frac{t}{l_\gamma}\right) \frac{\delta\rho}{\rho} \approx 10^4 \frac{\delta T}{T}}$$

for $t \sim 1/H$**.**

For turbulence we need

$$\delta \mathbf{T/T \sim 10^{-3} - 10^{-2}}$$

From the angular spectrum of CMBR

$$\delta T/T \leq 10^{-4}.$$

Is it possible that $\delta T/T$ is much larger at small scales, $l \sim l_{gal}$?

No observational bounds,
maybe(?) unnatural theoretically,
but not excluded.

Comments about current generation

The current is induced by pressure of photons diffusing from hotter regions to colder ones:

$$\dot{\mathbf{V}}_{\mathbf{d}} \sim \mathbf{F}_\gamma \mathbf{T} \sigma_{\mathbf{Th}}/\mathbf{m} \sim \mathbf{m}^{-3}$$

$\sigma_{\mathbf{Th}} = 8\pi\alpha^2/3\mathbf{m}^2$; $\mathbf{F}_\gamma$ **is the photon flux.**

Drift velocity $\mathbf{V_d = \dot{V} t_{free}}$,

$$\mathbf{t_{free}} = \frac{\mathbf{l_{free}}}{\mathbf{V_{thermal}}} = \frac{\mathbf{1}}{\sigma \mathbf{n}_\gamma} \sqrt{\frac{\mathbf{m}}{\mathbf{T}}} \sqrt{\frac{\mathbf{m}}{\mathbf{T}}} \sim \mathbf{m^3}$$

$\mathbf{V_d}$ is mass independent.

If only Thomson scattering is taken into account, no current can be induced by this mechanism - the same force and resistance.

Coulomb resistance is larger for p than for e. Allows generation of electronic current.

$$t_{Coul}(pp) = \frac{1}{\sigma_R n_p}\sqrt{\frac{m}{T}} \sim \frac{10^9}{T}\left(\frac{m}{T}\right)^{3/2}$$

where $\sigma_R \sim \alpha^2/q^2 \sim 1/mT$ - Rutherford cross-section; $n_p \sim 10^{-9} n_\gamma$ - much smaller number density of scatterers but much larger probability of interaction.

For protons: $t_{Coul} \ll t_{Th}$ and protons do not make current.

But remember about current conservation:

$$\mathbf{divJ} = \dot{\rho}$$

and quick neutralization by long-range Coulomb force, leading to $\rho = \mathbf{0}$.
Current loops can be created by vortical perturbations in photon distribution.

Electronic conductivity, estimated from:

$$\mathbf{m\dot{V}_d = eE}$$

and $\mathbf{V_d = eEt_{free}/m}$.

$$\kappa = \frac{\mathbf{3}}{\mathbf{2}\alpha}\frac{\mathbf{n_e}}{\mathbf{n}_\gamma}\frac{\mathbf{m_e^2}}{\mathbf{T}}$$

For high conductivity:

$$\partial_{\mathbf{t}}\mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\mathbf{1}}{\kappa}\nabla \times \mathbf{J}$$

Solution:

$$\mathbf{B} \sim \int_0^t \mathbf{dt_1} \left(\frac{\mathbf{2}\pi\,\mathbf{J}}{\lambda\,\kappa}\right) \mathbf{e}^{\mathbf{2}\pi\,\mathbf{v}\,\mathbf{t_1}/\lambda}$$

Exponential rise is not large:

$$\frac{\mathbf{2}\pi\mathbf{vt}}{\lambda} \approx \mathbf{500}\,\mathbf{T}_{\mathrm{eV}} \left(\frac{\delta\mathbf{T}}{\mathbf{T}}\right)_\lambda$$

Either larger $\delta\mathrm{T}/\mathrm{T}$ are necessary or generation of B at higher T - but it gives smaller scales.

$$\frac{\mathbf{B_0}}{\mathbf{T^2}} \approx \mathbf{10^3}\,(\mathbf{4}\pi\alpha)^{\mathbf{3/2}} \left(\frac{\mathbf{t}}{\lambda}\right) \left(\frac{\mathbf{l}_\gamma}{\lambda}\right)^{\mathbf{3}} \left(\frac{\mathbf{T}}{\mathbf{m_e}}\right)^{\mathbf{2}} \approx \mathbf{10^{-8}}\mathbf{T}^{\mathbf{3}}_{\mathrm{eV}}$$

where $\lambda = \mathbf{l}_\gamma$.

Adiabatic compression during galaxy formation by

$$\mathbf{B \sim 1/l^2 \approx (10^2)^2 = 10^4}.$$

Scale about 1 kpc.

Earlier formation - larger magnitude but smaller scales by $1/T^2$.

Chaotic line reconnection could create magnetic field at larger, galactic scale l_{gal}, but the magnitude of this field would be suppressed by $(l_B/l_{gal})^{3/2}$.

All scales give comparable contributions at l_{gal}.

VORTICITY GENERATION WITH LARGE FLUCTUATIONS

Start from kinetic equation:

$$(\partial_t + \mathbf{V} \cdot \nabla) f = I_{coll}$$

If I_{coll} is dominated by the elastic term, then integrating both parts over $\mathbf{d^3p/(2\pi)^3}$ we obtain the continuity equation:

$$\dot{n}(\mathbf{x}) + \nabla \mathbf{J} = 0$$

where J is the photon flux:

$$\mathbf{J} \equiv \mathbf{v} n = \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}}{E} f$$

and v is the average macroscopic velocity of the photon plasma. In the standard way the diffusion equation can be derived:

$$\dot{\mathbf{n}} = \mathbf{D}\, \boldsymbol{\Delta} \mathbf{n}$$

where $\mathbf{D \approx l_{free}/3}$.

One step further:

$$\frac{\partial}{\partial t}K_i(\mathbf{x},t) + 4H\,K_i(\mathbf{x},t) + \tau_w^{-1}K_i = S_i$$

where

$$\begin{aligned} S_i &= \frac{\partial}{\partial x^j}K_{ij}(\mathbf{x},t), \\ K_i &= \int k_i\ f_\nu(E,\mathbf{k};t,\mathbf{x})\frac{d^3\mathbf{k}}{(2\pi)^3}, \\ K_{ij} &= \int \frac{k_i k_j}{E}\ f_\nu(E,\mathbf{k};t,\mathbf{x})\frac{d^3\mathbf{k}}{(2\pi)^3}. \end{aligned}$$

K_{ij} **approximately satisfies diffusion equation.**

NB: $\nabla \times \mathbf{S} \neq 0$ for initial chaotic distribution of bubbles of excess of ν over $\bar{\nu}$ and vice versa:

$$\epsilon_{mli}\partial_l S_i = \epsilon_{mli} \int \frac{k_i k_j}{E} \partial_l \partial_j f_\nu(E, \mathbf{k}; t, \mathbf{x}) \frac{d^3\mathbf{k}}{(2\pi)^3}.$$

Some dissipation mechanism is necessary and it will be helpful to have multicomponent plasma with much different mean free path of constituents, as well as breaking of thermal equilibrium.

A MECHANISM FOR LARGE FLUCTUATIONS OF LEPTONIC CHARGE

Only one assumption: sterile neutrino weakly mixed with active one(s).
Temperature range 1-10 MeV.
Time interval 0.01-1 sec.
Constituents: three types of active neutrinos, photons, e^+e^-, a little protons and neutrons, $\sim 10^{-9}$.

Effective potential (refraction index):

$$\mathbf{V^a_{eff}} = \pm \mathbf{C_1} \eta \mathbf{G_F T^3} + \mathbf{C^a_2} \frac{\mathbf{G^2_F T^4 E}}{\alpha} \ ,$$

where $C \sim 1$ **and** η **is the plasma charge asymmetry:**

$$\eta^{(e)} = 2\eta_{\nu_e} + \eta_{\nu_\mu} + \eta_{\nu_\tau} + \eta_e - \eta_n/2$$

$$\eta^{(\mu)} = 2\eta_{\nu_\mu} + \eta_{\nu_e} + \eta_{\nu_\tau} - \eta_n/2 \ \text{(for } \nu_\mu\text{)}$$

η **is difference between numbers of particles and antiparticles.**

Kinetic equation for density matrix:

$$\dot{\rho} = \left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)\rho =$$

$$i\left[\mathcal{H}_m + V_{eff}, \rho\right] +$$

$$\int d\tau(\bar{\nu}, l, \bar{l})\left(f_l f_{\bar{l}} A A^+ - \frac{1}{2}\left\{\rho, A\bar{\rho}A^+\right\}\right) +$$

$$\int d\tau(l, \nu', l')\left(f_{l'} B\rho' B^+ - \frac{1}{2} f_l \left\{\rho, BB^+\right\}\right)$$

System of coupled nonlinear integro-differential equations with oscillating solutions.

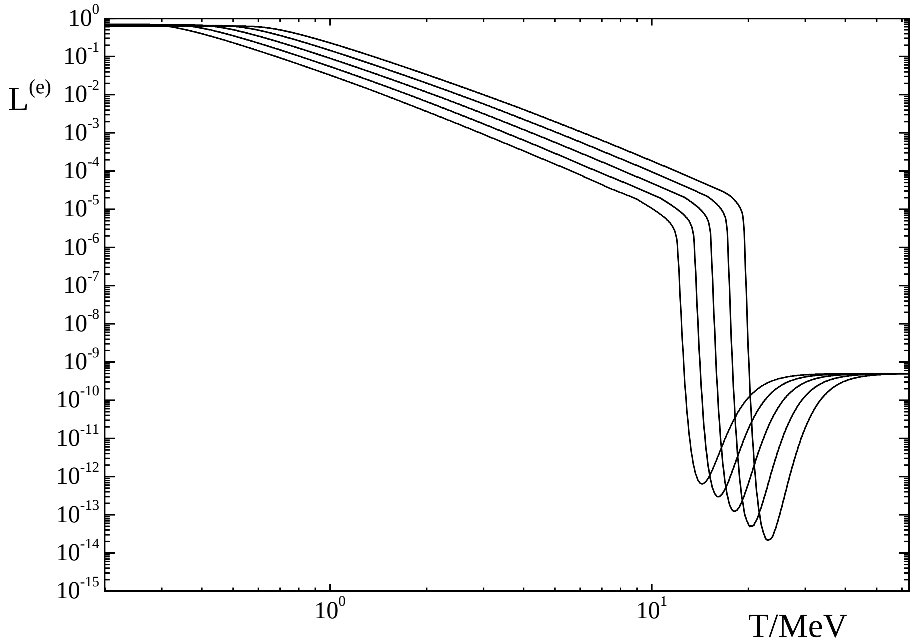
Effective potential depends upon charge asymmetry of neutrinos, i.e. upon the integral over all neutrino spectrum.

Very difficult to solve numerically, a lot of instabilities, non-physical chaotic oscillations (but not everybody agrees). Fortunately analytic solution is possible!

Resonance (MSW) transition, i.e. level crossing when

$$\delta \mathbf{E} + \mathbf{V}_{\mathbf{eff}} = \mathbf{0}$$

Resonance occurs at different T for ν and $\bar{\nu}$. One of the transitions into ν_s or into $\bar{\nu}_s$ is more favored.
It changes charge asymmetry and leads to exponential rise or decrease of the lepton asymmetry. The sign of the effect is energy dependent.



Evolution of leptonic charge as a function of T or t.

In the minimum the sign of the effect is determined by the sign of baryonic charge fluctuations and the asymptotic value of L may have either sign.

Inhomogeneous case - charge asymmetry should be a little inhomogeneous:

$$\delta\eta/\eta \sim \mathbf{10^{-5} - 10^{-3}}$$

The first number is certain, the second is allowed.

The evolution of the neutrino asymmetry is described by the equation:

$$\dot{\mathbf{L}}_{\nu_\alpha}(\tilde{\mathbf{x}}, \mathbf{t}) = \mathbf{a}(\mathbf{t}) \left[2\, L_{\nu_\alpha}(\vec{x}, t) + \bar{L} + \delta B(\vec{x})\right] + \mathbf{D}(\mathbf{t}) \nabla^{\mathbf{2}} \mathbf{L}_{\nu_\alpha}(\tilde{\mathbf{x}}, \mathbf{t})$$

where the initial background asymmetry is

$$\tilde{\mathbf{L}} = \bar{\mathbf{L}} + \delta \mathbf{B}(\mathbf{x}),$$

D(t) is the diffusion coefficient, and the function a(t) is initially negative and generates an exponential decrease of the asymmetry, but at some critical time t_c it changes sign.

Positive a(t) creates a huge rise of the asymmetry, but while a(t) is negative, the asymmetry drops down to a very small value. In a more accurate formulation a(t) would also depend on the asymmetry itself, but in what follows we are interested in rather small values of the asymmetry, where non-linear effects are not important.

Solution:

$$
\begin{aligned}
L_{\nu_\alpha}(x,t) \; = \; & \bar{L} \int_{t_{\rm in}}^{t} dt'\, a(t')\, e^{2\int_{t'}^{t} dt''\, a(t'')} + \\
& + \; e^{2\int_{t_{\rm in}}^{t} dt'\, a(t')} \int d^3k\, e^{i\vec{k}\vec{x}}\, \hat{L}_{\nu_a}(\vec{k}, t_{\rm in}) \\
& * \; e^{-k^2 \int_{t_{\rm in}}^{t} dt'\, D(t')} + \\
& + \int_{t_{\rm in}}^{t} dt'\, a(t')\, e^{2\int_{t'}^{t} dt''\, a(t'')} \\
& * \int d^3k\, e^{i\vec{k}\vec{x}}\, \delta\hat{B}(\vec{k}, t_{\rm in}) \\
& * \; e^{-k^2 \int_{t'}^{t} dt''\, D(t'')}
\end{aligned}
$$

The first term in this expression can be explicitly integrated:

$$(\mathbf{1/2})\bar{\mathbf{L}}\left[\exp\left(\mathbf{2}\int_{\mathbf{t_{in}}}^{\mathbf{t}}\mathbf{dt_2 a}(\mathbf{t_2})\right)-\mathbf{1}\right]$$

giving a rising term (after some initial decrease) plus a constant initial value of $\bar{\mathbf{L}}$.

The second term can be also integrated because the initial value $\hat{\mathbf{L}}(\mathbf{k}, \mathbf{t}_{\rm in})$ is supposed to be homogeneous and so its Fourier transform is just delta-function, $\delta^3(\mathbf{k})$. The integral gives

$$\mathbf{L}_{\nu_a}^{(\mathbf{in})} \exp\left(\mathbf{2}\int_{\mathbf{t_{in}}}^{\mathbf{t}} \mathbf{dt_2 a(t_2)}\right)$$

the same falling-then-rising term.

The third (oscillating) term. To evaluate the integral let us substitute:

$$\delta\hat{\mathbf{B}}(\tilde{\mathbf{k}}, \mathbf{t}_{\rm in}) = \int \mathbf{d^3x_1 e^{i\tilde{k}\tilde{x}_1}}\, \delta\mathbf{B}(\tilde{\mathbf{x}}_1)$$

where $\delta B(\vec{x}_1)$ is the initial value of the inhomogeneous term. Now we can integrate over d^3k. We have the integral of the type

$$\int \mathbf{d^3k}\, \exp[-\mathbf{S^2k^2} + \mathbf{i\tilde{k}}(\tilde{\mathbf{x}} - \tilde{\mathbf{x}}_1)]$$

the scalar product of vectors $\vec{k}$ and $\vec{r} = \vec{x} - \vec{x}_1$ is equal to $\vec{k}(\vec{x} - \vec{x}_1) = kr\cos\theta$, and

$$\mathbf{S^2(t)} = \int_{\mathbf{t_{in}}}^{\mathbf{t}} \mathbf{dt_2 D(t_2)}$$

Integration over angles in
$\mathbf{d^3k = 2\pi k^2 dk d}(\cos\theta)$
is trivial, it gives $\sin\mathbf{kr}/\mathbf{kr}$**.**
The remaining integration can be done as follows:

$$\int \mathbf{dkk}\sin\mathbf{kr}\exp[-\mathbf{S^2k^2}] =$$

$$(\mathbf{d}/\mathbf{dr})\int \mathbf{dk}\cos\mathbf{kr}\exp[-\mathbf{S^2k^2}]$$

The remaining integration can be performed if we expand the range of integration from minus to plus infinity. Introducing a new variable $\vec{x}_1 = \vec{x} - S(t_1)\vec{\rho}$ we finally obtain

$$\int \mathbf{dt_1 a(t_1)}\, \mathbf{e}^{\mathbf{2}\int_{\mathbf{t_1}}^{\mathbf{t}} \mathbf{dt_2 a(t_2)}}$$

$$\int \mathbf{d^3}\rho\, \delta\mathbf{B}(\mathbf{x} - \mathbf{S}(\mathbf{t_1})\rho)\, \mathbf{e}^{-\rho^{\mathbf{2}}}$$

This is the contribution the lepton asymmetry L_{ν_a} generated by the (small) baryonic inhomogeneities.

Its asymptotic rise at large t is similar to the rise of other terms, but its exponential decrease at intermediate stage could be considerably milder and this small term could become larger than the homogeneous one.

As a result, this term could become dominant with the sign determined by the sign of the fluctuations in the baryon asymmetry. We can see this in a simple example assuming that the function $a(t)$ has the form

$$\mathbf{a(t) = a_1(t - t_c)}$$

and that the fluctuations of the asymmetry are described by one harmonic mode:

$$\delta \mathbf{B}(\mathbf{x}) = \epsilon_{\mathbf{B}} \cos \mathbf{k_0 x}.$$

The result:

$$\delta L(\vec{x}) = a_1 \epsilon_B \cos \vec{k}_0 \vec{x} \; e^{a_1 (t-t_c)^2 - S^2(t)\, k_0^2}$$
$$\left[\int_{t_c - t_{in}}^{t-t_c} dt_1 t_1 e^{-a_1 t_1^2 + S^2(t_1)\, k_0^2} + \right.$$
$$\left. \int_0^{t_c - t_{in}} dt_1 t_1 e^{-a_1 t_1^2} \left(e^{S^2(t_1)\, k_0^2} - e^{-S^2(t_1)\, k_0^2} \right) \right]$$

Both terms rise as $\exp[a_1(t-t_c)^2]$**, i.e. in the same way as the other homogeneous terms (we assume that** $S(t)$ **is finite at large** t **and not too large). The first term is exponentially suppressed as** $\exp[-a_1(t_c - t_{in})^2]$ **also at the same level as the homogeneous terms.**

The second term, which vanishes in the homogeneous case ($k_0 = 0$ or $S = 0$) is not exponentially suppressed. In the limit of a large a_1 the integral can be evaluated as $\sim S^2(0)\, k_0^2/a_1$. It is small but not exponentially small. Thus, one can easily imagine a situation when the last term dominates and the resonance enhancement of lepton asymmetry in the background of small fluctuations of baryon asymmetry could create domains with large and different lepton asymmetry.

Leptonic domain structure with almost flat spectrum is created.

Neutrino streaming from ν-rich regions pushes electrons and positrons but with different efficiency and creates electric current J, with $\nabla \times \mathbf{J} \neq 0$.
The process took place during relatively short period, when

$$\mathbf{l}_\nu \sim \mathbf{t}_\mathbf{U}.$$

If $l_\nu < t_U$, diffusion is slow,
if $l_\nu > t_U$, interaction with electrons is weak.

At large wave lengths the spectrum is cut-off by neutrino decoupling scale.

At small wave lengths it is cut-off by diffusion damping.

Electric current is created because of different interaction of neutrino with electrons and positrons.

Reynolds number is of the order of unity,

but initial vorticity is non-zero.

Possibly turbulence might develop under these conditions(?).
Do we need it?

Gravitational waves, potentially observable by LISA, might be generated.

Spectrum of turbulence.

Direct cascade - smaller eddies but with larger R from low R eddies at larger scales, is it possible? Usually - opposite.

Rate of energy transition to scale k:

$$\varepsilon_{\mathbf{k}} \equiv \frac{\mathbf{1}}{(\rho + \mathbf{p})} \frac{(\mathbf{d}\rho_{\mathbf{k}})_{\rm in}}{\mathbf{dt}} \sim \frac{(\mathbf{du_k^2})_{\rm in}}{\mathbf{dt}}$$

Usual result (see e.g. L&L) for quasistationary process:

$$\varepsilon_{\mathbf{k}} \sim \mathbf{u_k^3 k} \ ,$$

External source:

$$\frac{\mathbf{d}\varepsilon_{\mathbf{k}}}{\mathbf{dk}} = \mathbf{P}_{\mathrm{ext}}(\mathbf{k})$$

This is different from conventional Kolmogorov type turbulence where energy is supposed to be injected at a single scale and $\varepsilon_k = \varepsilon$ is momentum independent.

Define energy spectrum:

$$\langle \mathbf{u}^{\mathbf{2}}(\mathbf{x}) \rangle \equiv \int_{\mathbf{0}}^{\infty} \mathbf{E}(\mathbf{k})\mathbf{dk}$$

For power law external source:

$$\mathbf{E(k) \sim \varepsilon_S^{2/3}\, k_S^{-\gamma}\, k^{\gamma-5/3}}\ .$$

Kolmogorov spectrum corresponds to $\gamma = 0$.

Velocity spectrum:

$$\mathbf{u_k \sim \left(\frac{\varepsilon_k}{k}\right)^{1/3} = u_S \left(\frac{k}{k_S}\right)^{\gamma/2-1/3}} \quad ,$$

where $u_S = (\varepsilon_S/k_S)^{1/3}$, $1/k_S$ is the largest length scale at which energy is injected into turbulent motions, $u_S = (\varepsilon_S/k_S)^{1/3}$.

Relativistic motion!?

If $\gamma > 2/3$ the characteristic velocity u_k increases with k and u_k may reach 1 at $k = k_{rel}$ - very interesting for gravitational waves.
However, spectrum of neutrino domains is not peaked at high k and the effects are minor. Most optimistic expectation are near expected sensitivity of LISA.
Possibly more efficient: neutrino domains formed by Affleck-Dine leptogenesis. Not unnatural but not as simple as $nu_a - \nu_s$ oscillations. Maybe some enhancement can be found?

Conclusion about magnetic fields.

Neither of discussed above models may explain the observed galactic fields without galactic dynamo.

Intergalactic fields $\mathbf{B_{ig} \sim 10^{-3} B_{gal}}$**, scale:** $\sim (\mathbf{0.1 - 1})$ **Mpc. They may have a common origin with galactic ones. Adiabatic compression:** $B \sim 1/l^2$**:**

$$\mathbf{l_{gal}^{(in)}/l_{gal} \sim 10^2}, \quad \mathbf{l_{ig}^{(in)}/l_{ig} \sim 3},$$

Expect $B_{gal} \sim 10^3 B_{ig}$**, if common origin and no galactic dynamo amplification.**

Possible and desirable galactic dynamo amplification by $10^{15\pm5}$!

If this is the case then primordial magnetic fields would not influence CMBR polarization. Otherwise, if $B \sim 10^{-9}$ Gauss, the Faraday rotation may be observable.

Maybe magnetic field generation at the epoch of large scale structure formation is more favorable: large vorticity, reioniztion,... but difficult to calculate. Existing estimates give low results.